Efficient Estimation of Asset Pricing Models with Product Variety and Production Data*

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Abstract

We use real and financial restrictions from a dynamic multisector production-based asset pricing model and U.S. manufacturing data for GMM estimation of salient parameters governing recursive CES preferences. Industry investment and production data provide strong instruments due to high autocorrelations. We obtain reasonable, efficient estimates of risk-aversion and IES. In contrast, we obtain weak identification using only asset market restrictions with returns and aggregate consumption as instruments. Identification diagnostics indicate that modeling product variety, real structural restrictions, and production-based instruments each contribute to strong identification. We quantitatively show a strong, negative relation of product elasticity of substitution and equity risk-premiums.

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1 Introduction

A long-standing literature estimates risk aversion and the intertemporal elasticity of substitution in consumption (IES) using GMM and data on aggregate (or market level) asset returns (Hansen and Singleton (1982, 1983), Epstein and Zin (1991)). Typically, lagged consumption and (aggregate) returns are utilized as instrumental variables (IVs). But it is well known that these IVs are weak because of low autocorrelation in returns as well as low cross-autocorrelations in consumption and returns, i.e., these tests suffer from a weak identification problem (Stock and Wright (2000)). Our study utilizes an empirical test design with two novel features relative to the literature: We model consumer preferences for product variety, and we exploit highly autocorrelated industry production data that serve as strong IVs. Utilizing a multisectoral production-based asset pricing model with constant elasticity of substitution (CES) consumer preferences over product variety, we obtain robust and efficient estimation of salient parameters governing consumer preferences: risk aversion, IES, and the product elasticity of substitution (ES). The estimated parameters are economically appealing. We find that adding real structural restrictions from industry production and investment equilibrium paths, as well as incorporating preferences over product variety, helps explain the strong identification of risk aversion and IES (along with the ES).[1]

At the microeconomic level, agents choose consumption bundles (or baskets) of various types of consumption goods produced by firms in different sectors, which is addressed by existing microeconomic theory (Arrow and Hahn (1971)). Shareholder value maximization then implies that the production decisions of public firms will reflect the preferences of their equity owners who are also consumers. Hence, in a realistic multigood, or multisector context, parameters describing consumer preferences should be reflected not only in aggregate

[1] While our multisector model considers consumer goods as well as intermediate goods sectors, for expositional parsimony we focus on consumer goods industries to facilitate comparisons with the received asset pricing and applied economics literature.

consumption and asset returns but also in industry-specific production information,[2] potentially enhancing empirical identification. The results verify that this intuition has significant empirical content: Our estimation approach significantly improves identification of structural parameters describing consumer preferences, relative to benchmark estimations using equity returns and aggregate consumption data alone.

We use GMM with heteroskedasticity- and autocorrelation-consistent inference. For this estimation, we utilize industry data on capital, investment, materials inputs, and industry productivity from the NBER-CES database of U.S. manufacturing industries. As we indicated above, an important aspect of the industry production data is the high levels of own and cross-autocorrelations in the endogenous variables, i.e., investment and material inputs. This is consistent with the literature that documents high short run predictability of capital investment (Eberly, Rebelo and Vincent (2012)). Thus, in contrast to the weak IV problem with asset returns and consumption growth, using lagged industry production inputs as IVs allows stronger correlation with the optimality conditions related to firms' investment and input choices, and hence stronger identification of the structural parameters.

Our estimation methodology yields strong joint identification of risk aversion, IES and ES. The point estimates are statistically significant and have low dispersion across different specifications—or choice of IVs—and are economically appealing. The risk aversion estimate is clustered around 5, well within the range of risk aversion values considered reasonable in the literature (Mehra and Prescott (1985)). The IES estimates cluster around 0.2 and the tests strongly reject the null of zero IES. The literature reports a wide range of estimates for IES. A long-standing literature finds IES in the low range using aggregate data (Hall (1988), Epstein and Zin (1991), van Binsbergen et al. (2008)). More recently, using a novel structural estimation approach, based on "mortgage notches" in the UK, Best et al.

[2] This applies to general equilibrium production models with complete markets (Cochrane (1991)) or with incomplete markets (Horvath (2000)). More generally, even with agency problems due to separation of ownership and control (Berle and Means (1932), Jensen and Meckling (1976)), the preferences of equity owners are not irrelevant for managers.

(2020) also report estimates of IES around 0.1, while a meta-analysis of IES estimates in the literature finds estimates clustered between 0.3-0.4 (Havránek (2015)).[3] The estimates of risk aversion and IES also imply that consumer risk preferences are not statistically distinct from the power expected utility specification. Finally, the estimates of ES for manufactured consumption goods cluster around 2, which is consistent with the estimates in the literature for manufactured goods using import data (Broda and Weinstein (2006)).

In light of these estimation results, we investigate the contributions to improved identification of explicitly recognizing (or modeling) consumer preferences over product variety (through the ES); utilizing real restrictions, i.e., the intertemporal capital investment and intratemporal materials input optimality conditions; and using investment and production data in IVs. We find that structural restrictions added by preferences for variety—through the ES—aids identification of the representative consumer's risk aversion and IES. The analysis also shows that identification of risk aversion and IES deteriorates sharply if we "eliminate" the real optimality conditions. Consistent with intuition, our analysis indicates that the intertemporal investment Euler condition contributes significantly to the identification of risk aversion and IES, whereas the intratemporal materials input condition contributes significantly to the identification of ES. Consequently, both the real restrictions contribute to the identification of the parameters of interest.

Similarly, estimates of risk aversion and IES are no longer statistically significant, and model performance deteriorates, when we do not use production data in IVs. These diagnostics are consistent with the view that investment and production data provide strong IVs for identification in asset pricing models.

Overall, our analysis strongly suggests that consumer preferences for product variety—parameterized in our model by the ES—significantly affect the equilibrium risk premium.

[3] However, another strand of the literature reports estimates of IES in excess of 1 (see Bansal and Yaron (2004)).

Hence, making the ES explicit in the model's equilibrium conditions imposes additional structural restrictions on returns, compared with models where the role of the ES is suppressed. It is, thus, important to examine whether and how the ES affects equilibrium risk premium. Indeed, the relation of consumer preferences for product variety and equity risk premium is of independent interest, but this topic is relatively under-explored. Our framework allows a quantitative examination of how references for variety, risk aversion, and the IES jointly affect industry-level equity risk premium (ERP) that attract substantial interest in the literature (e.g., Fama and French (1997)) and the investment industry.

There is an intuition that, at the industry level, ERP should ceteris paribus be positively related to product differentiation. This is because commodities and basic consumption goods with low product differentiation exhibit small cyclical variation in price-cost markups and, hence, profits. In contrast, highly differentiated goods have relatively large cyclical variation in profit margins, amplifying the negative covariation between the stochastic discount factor and returns. But, conceptually and empirically, goods with low product differentiation have greater demand price elasticity compared with highly differentiated goods (Berry et al. (1995), Broda and Weinstein (2006)). Because the ES captures demand price elasticity in the CES setting, intuitively ES and ERP should be negatively related.

Our quantitative analysis indeed verifies that ES and ERP are negatively related. That is, differentiated goods ceteris paribus have higher ERP relative to non-differentiated goods. Intuition also suggests that more risk averse investors or those with lower IES (when IES is less than one) will demand greater risk premium due to the higher procylicality of profits of goods with inelastic demand. And this is confirmed by our analysis. Because of these interaction effects, sectoral ERP can be sizeable for medium risk aversion and low IES—consistent with our point estimates for these parameters—in industries with low demand elasticity; conversely, the ERP can be low even with relatively large risk aversion in high demand elasticity industries. Therefore, variations in product characteristics measured through their price elasticities are consistent with cross-sectional dispersion in industry ERP that is observed in the data.

To our knowledge, our study is the first to show that structural restrictions from investment and production equilibrium paths in multi-good settings, and strong IVs offered by highly autocorrelated industry variables, allow robust and efficient estimation of structural parameters describing consumer preferences. We point out that explicit recognition of consumer preferences for variety (through the ES) and utilization of industry-level real equilibrium restrictions—along with the attendant use of production data—are together needed to improve efficient estimation of risk aversion and IES, parameters that are of particular interest to financial economists. Notably, a growing literature estimates ES using imports and consumption data (Feenstra (1994), Broda and Weinstein (2006), Redding and Weinstein (2020)). By using moment conditions in our multisector model and utilizing sectoral production, investment, and asset return data, we present a novel estimation framework for the *joint* estimation of risk aversion, IES and ES through GMM.

We add to the literature on production-based asset pricing models (Cochrane (1991), Jermann (1998)), in particular to the growing strand on multisector general equilibrium models (Papanikolaou (2011), Kogan and Papanikolaou (2013, 2014), Doshi and Kumar (2025)). While the existing literature typically focuses on implications of such models for asset prices, our study is among the first to utilize such models for efficient estimation of parameters governing consumer preferences over stochastic consumption baskets, with industry-level production data. Furthermore, we exploit the multisector setting to quantitatively examine the interaction of risk aversion, IES, and ES on (industry-level) equity risk premiums. We highlight the importance of considering consumer preferences for product variety in helping explain variations in observed industry-level risk pemiums.

Our quantitative analysis is linked especially to the literature that modifies the canonical aggregate asset pricing model to help resolve the equity premium puzzle. One strand of this literature argues that consumption volatility measured from National Income and Product Accounts (NIPA) may not correctly represent the actual consumption risk faced by investors. For example, Ait-Sahalia, et al. (2004) observe that NIPA consumption weights focus on basic consumption goods. By incorporating luxury good consumption through nonhomothetic preferences, they are able to explain observed equity premium at relatively low levels of risk aversion; hence, our analysis is consistent with their results. Meanwhile, Parker and Juillard (2005), Savov (2011) and Kroencke (2017) point out other sources of mis-measurement of consumption risk. Another strand of the literature emphasizes the role of long run risks in exchange and production economies (Bansal and Yaron (2004), Croce (2014)). Our study suggests that the *composition* of the aggregate consumption basket—in terms of demand price elasticities of component goods—is a significant determinant of aggregate consumption risk.

Finally, our results complement findings in other areas of finance where richer parameterization of consumer preferences and using novel data improves identification. For example, Huang and Shaliastovich (2014) show that using recersive preferences—in particular, imposing parameteric restrictions for early resolution of uncertainty—and using equity options data help identify objective probabilities and risk adjustments.

The paper is organized as follows. Section 2 describes the model. Section 3 undertakes estimation of the model. Section 4 analyzes the determinants of estimation efficiency. Section 5 quantitatively examines the relation of preferences for product variety to the equity risk premium. Section 6 summarizes the analysis and concludes.

2 A Multi-Sector General Equilibrium Model

In a discrete time, infinite horizon setting, we consider an economy consisting of J competitive sectors (or industries), each composed of a continuum of identical firms of unit mass, and producing a different good. It is therefore notationally convenient to exposit the model at the sectoral level. Sectors are partitioned into J_c sectors that produce (final) consumption goods and sectors that produce two types of intermediate goods: J_h sectors that produce material inputs for production and J_k sectors that produce capital inputs.

2.1 Consumption and Portfolio Investment

There is a continuum of identical consumer-investors (CI) in the economy; the number of CIs is normalized to unity, without loss of generality. The (representative) CI's income each period comprises of dividend payouts from firms, net changes in the value of her/his stock portfolio, and income from a riskless security. Time is discrete At each t, the CI chooses the consumption vector $\mathbf{c}_t = (c_{1t}, ...c_{Jc,t})$, taking as given the corresponding vector of consumption good prices $\mathbf{p}_t^c$, with the first consumption good serving as the numeraire, without loss of generality.

Firms are unlevered, publicly owned and their equity ownership trades in frictionless security markets. The CI also has access every period to a (one-period) risk-free security (f) that pays a unit of the numeraire good next period. The mass of risk-free securities is fixed at unity. The CI's asset holdings at the beginning of the period are denoted by the $(J+1)-$dimensional vector $\mathbf{q}_t$. Along with consumption, the CI simultaneously chooses her/his new asset holdings $\mathbf{q}_{t+1}$, taking as given the corresponding (ex-dividend) asset prices $\mathbf{s}_t$. The dividend payouts per share are denoted by $\mathbf{d}_t$ (with the risk-free asset payouts fixed at 1).

The representative CI has Epstein and Zin (1989) preferences over intertemporal streams of consumption bundles $\{C_t\}_{t=1}^{\infty}$ that are expressed in recursive form as

$$\mathcal{U}_t = \left[(1-\alpha)C_t^{1-\eta} + \alpha \mathbb{E}_t \left[\mathcal{U}_{t+1}^{1-\gamma}\right]^{\frac{1-\eta}{1-\gamma}}\right]^{\frac{1}{1-\eta}}, \tag{2.1}$$

when $\gamma \neq 1$ and $\eta \neq 1$. In (2.1), C_t is the aggregated consumption index with constant elasticity of substitution (CES) among consumption goods, that is,

$$C_t = \left[\sum_{j=1}^{J_c} \phi_j (c_{jt})^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \tag{2.2}$$

where $\sigma > 1$ is the ES and $0 < \phi_j < 1$ are the utility weights; α controls the subjective rate of impatience; γ determines the degree of risk aversion; and η^{-1} measures the intertemporal elasticity of substitution (IES) over consumption baskets.

The CI's budget constraint is given by

$$\mathbf{p}_t^c \cdot \mathbf{c}_t \leq \mathbf{q}_t \cdot (\mathbf{d}_t + \mathbf{s}_t) - \mathbf{q}_{t+1} \cdot \mathbf{s}_t \equiv W_t. \tag{2.3}$$

Because preferences are strictly increasing, the budget constraint (2.3) will be binding in any optimum and hence W_t also represents the total consumption expenditure at t. In the standard fashion for Dixit and Stiglitz (1977) preferences, intratemporal optimization yields the consumption demand functions (see the Online appendix)

$$c_{jt}(\mathbf{p}_t^c, W_t) = \frac{W_t}{P_t} \left[\frac{P_t \phi_j}{p_{jt}} \right]^{\sigma} \tag{2.4}$$

where $P_t = \left[\sum_{j=1}^{J_c} (\phi_j)^{\sigma} (p_{jt})^{1-\sigma} \right]^{1/(1-\sigma)}$ is the aggregate price index. At the optimum, the aggregate real consumption $C_t \equiv C(\mathbf{c}_t) = \frac{W_t}{P_t}$, which is the real income.

Meanwhile, the CI's portfolio optimization equates the real current security price to expected present value of real equity payoffs next period. The real SDF (or pricing kernel) is the intertemporal marginal rate of substitution of real consumption (IMRS). Letting $\Lambda_t \equiv \frac{\partial \mathcal{U}_t}{\partial \mathcal{C}_t} = (1-\alpha) C_t^{-\eta} \mathcal{U}_t^{\eta}$ denote the marginal valuation at t, the SDF for the one-period investment horizon is $\Lambda_{t,t+1} \equiv \frac{\Lambda_{t+1}}{\Lambda_t}$. Using $C_t = \frac{W_t}{P_t}$ and following Epstein and Zin (1989), the real SDF can be written

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \alpha^\theta \left(\frac{G^W_{t+1}}{G^P_{t+1}}\right)^{-\eta\theta} R^{\theta-1}_{C,t+1}, \theta \equiv \frac{1-\gamma}{1-\eta}, \tag{2.5}$$

where G^W_{t+1} and G^P_{t+1} are the gross growth rates in aggregate income and the price index between t and $t+1$, respectively, and $R_{C,t+1}$ is the gross one-period (real) return on an asset that pays aggregate consumption as its dividend.[4] Hence, asset prices satisfy

$$\frac{\mathbf{s}_t}{P_t} = \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{\mathbf{d}_{t+1}+\mathbf{s}_{t+1}}{P_{t+1}}\right),\right] \tag{2.6}$$

which can be expressed in more conventional (or nominal terms) by defining the one-period ahead *nominal* SDF $M_{t,t+1} \equiv P_t\left(\frac{\Lambda_{t,t+1}}{P_{t+1}}\right)$. Since the one-period return between t and $t+1$ in each sector is $R_{j,t+1} = \left(\frac{d_{j,t+1}+s_{j,t+1}}{s_{j,t}}\right)$, and the return on the one-period nominally riskless bond is $R_{f,t+1} = (1/s_{f,t})$, in the standard fashion Equation (2.6) can be written as the restriction

$$\mathbf{1} = \mathbb{E}_t\left[M_{t,t+1}\mathbf{R}_{t+1}\right], \tag{2.7}$$

where $\mathbf{1}$ and $\mathbf{R}_{t+1}$ are the unit and gross nominal returns vectors, respectively.

2.2 Production and Dividends

The representative firm in the typical sector produces output Y_t through the production function:[5]

$$F(K_t, H_t, A_t) = A_t(K_t)^{\psi_K}(H_t)^{\psi_H}, \tag{2.8}$$

where K_t is the firm's capital stock at the beginning of t; H_t are materials input chosen during the period; A_t represents a stochastically evolving industry-wide productivity level;

[4] More precisely, $\frac{\Lambda_{t+1}}{\Lambda_t} = \left(\frac{G^W_{t+1}}{G^P_{t+1}}\right)^\eta \left(\frac{U_{t+1}}{\mathbb{E}_t[U^{1-\gamma}_{t+1}]^{\frac{1}{1-\gamma}}}\right)^{\eta-\gamma}$, which can be shown to equal (2.5).

[5] For notational ease, we suppress subscripts for sectors and firms unless necessary for exposition.

and $\psi_K \in (0,1), \psi_H \in (0,1)$ are the output elasticities of capital and inputs, respectively. The sectoral productivity shocks follow a first order log-autoregressive stochastic process, that is,

$$a_t = \rho_a a_{t-1} + \varepsilon_{at}, \tag{2.9}$$

where ε_{at} is a normal mean zero variable with a stationary variance-covariance matrix (across sectors) Φ_a. Capital stock K_t evolves according to

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{2.10}$$

where δ is the per-period depreciation rate and I_t is the investment at t.

Similar to the literature (Kiyotaki (1988), Horvath (2000)), we assume that firms in each sector combine intermediate goods to form a composite material input and investment good using the sector-specific CES functions:[6]

$$H_t = \left[\sum_{n=J_c+1}^{J_h} \varphi_{nj}^h (H_{jn,t})^{(\zeta_j^h - 1)/\zeta_j^h} \right]^{\zeta_j^h/(\zeta_j^h-1)} ; I_t = \left[\sum_{n=J_h+1}^{J} \varphi_{nj}^k (I_{jn,t})^{(\zeta_j^k - 1)/\zeta_j^k} \right]^{(\zeta_j^k-1)/\zeta_j^k}, \tag{2.11}$$

where $H_{jn,t}$ is the quantity of material intermediate good purchased by sector j from sector $n = J_c + 1, ..., J_h$; φ_{nj}^h is the sector-specific weight of this good, and $\zeta_j^h \geq 1$ is the ES among material intermediate goods in sector j. Analogously, one interprets $I_{jn,t}$, φ_{nj}^k and ζ_j^k for investment intermediate goods. The costs of material and investment intermediate goods are $\Upsilon_{jh,t} = \sum_{J_c+1}^{J_h} p_{nt} H_{jn,t}$ and $\Upsilon_{jk,t} = \sum_{J_h+1}^{J} p_{nt} I_{jn,t}$, respectively.

Sectors choose intermediate goods in a two-step process. In the first step, H_t and I_t are determined; in the second stage, conditional on (H_t, I_t), the individual intermediate goods H_{nt} and I_{nt} are chosen to minimize the intermediate cost expenditures Υ_{ht} and Υ_{kt}. This

[6]The counting notation in (2.11), the J sectors are partitioned as $\{1, ..., J_c; J_c + 1, ..., J_h; J_h + 1..., J\}$.

process yields the demand function for intermediate goods as (see the Online appendix):

$$H_{nt} = (\varphi_{nj}^h)^{\zeta_j^h} \left[\frac{p_{nt}}{X_t}\right]^{-\zeta_j^h} H_t; I_{nt} = (\varphi_{nj}^k)^{\zeta_j^k} \left[\frac{p_{nt}}{Z_t}\right]^{-\zeta_j^k} I_t, \tag{2.12}$$

where $X_t = \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_{nt})^{1-\zeta_j^h}\right]^{1/(1-\zeta_j^h)}$ and $Z_t = \left[\sum_{n=J_h+1}^{J} (\varphi_{nj}^k)^{\zeta_j^k} (p_{nt})^{1-\zeta_j^k}\right]^{1/(1-\zeta_j^k)}$ are the materials and investment intermediate goods price indices for sector. It can be shown that the effective composite material input demand H_t is such that $X_t H_t = \Upsilon_{ht}$, and similarly $Z_t I_t = \Upsilon_{kt}$. Apart from the costs of investment goods, firms are subject to convex capital adjustment costs so that the total investment cost function is

$$O(I_t, K_t) = Z_t I_t + 0.5\upsilon \left(\frac{I_t}{K_t}\right)^2 K_t, \tag{2.13}$$

where υ is the sector-specific capacity adjustment cost parameter. The process for determination of (H_t, I_t) will be specified below in the characterization of equilibrium.

The number of shares outstanding in each sector at the beginning of t is denoted by Q_t. Net cash flows are paid out as dividends. Then payouts from sector at t are

$$D_t = p_t Y_t - X_t H_t - O(I_t, K_t). \tag{2.14}$$

Dividends can be negative, which are financed by equity issuance.

2.3 Equilibrium

The state vector for firms in the typical sector at beginning of t is $\Omega_t = (W_t, P_t, A_t, X_t, Z_t, K_t)$; the first four elements of this vector are taken as exogenous by firms, while K_t and q_t are dynamically endogenous. At every t, conditional on Ω_t, the representative firm in each sector is instructed by shareholders to choose $\{H_{t+\tau}, I_{t+\tau}\}_{\tau=0}^{\infty}$ to maximize the conditional present value of real dividends given by[7]

$$\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} \left(\frac{D_{t+\tau}}{P_{t+\tau}} \right) \right], \tag{2.15}$$

subject to the production and capital accumulation constraints specified above.

In equilibrium, firms follow optimal investment and input choice strategies taking as given the prices for the industry good p_t, while consumers follow optimal consumption and portfolio policies represented by (2.4) and (2.6). The model is closed by the requirement that product and asset markets clear. It is convenient, from the viewpoint of our empirical analysis, to express firms' objective functions in nominal terms. Defining the nominal SDF for future payoffs at t as $M_{t,t+\tau} \equiv P_t \left(\frac{\Lambda_{t,t+\tau}}{P_{t+\tau}} \right), \tau = 0, 1, ..,$ the objective function in (2.15) can be re-expressed as $\mathbb{E}_t \left[\sum_{\tau=0}^{\infty} M_{t,t+\tau} D_{t+\tau} \right]$. Since $\Lambda_{t,t+\tau} \equiv \frac{\Lambda_{t+\tau}}{\Lambda_t}$ by construction, it will follow that $M_{t,t+\tau} = \frac{M_{t+\tau}}{M_t}, \tau = 0, 1, 2...(M_{t,t} = 1)$. Then, using the Bellman representation, we can define the nominal *cum-dividend* value function of the representative firm along the equilibrium path recursively as a function of the state:

$$V_t(\Omega_t) = \max_{I_t, H_t \geq 0} D_t + \mathbb{E}_t \left[V_{t+1}(\Omega_{t+1}) \right]. \tag{2.16}$$

The ex-dividend value of the firm will be denoted S_t.

2.4 Equilibrium Characterization for Consumer Goods

We will focus on the asset pricing implication of consumer goods industries (sectors $j = 1, ..., J_c$) for reasons of space and expositional parsimony. There are two principal reasons for this focus. First, the existing empirical asset pricing literature mostly focuses on ERP driven by the (direct) consumption of the representative consumer, so it facilitates intuition on the

[7] In general, there will not exist complete contingent markets in this model; hence, the discount rate is given by the representative consumer's marginal utility of real consumption (Brock 1982; Horvath 1998).

determinants of sectoral ERP—through comparisons of our results with the literature—to focus on consumer goods industries. Second, the demand functions for consumer goods—and, hence, their sales and dividends—involve the intratemporal ES and sectoral taste parameters (see (2.4)); these parameters have been estimated by a long literature, which provides a useful benchmark for checking the "reasonableness" of our estimates. In contrast, there is sparse literature on the estimation of the sector-specific CES production and investment parameters ζ^h and ζ^k that drive the demand functions of intermediate goods producers.[8]

We now present the optimality conditions for the representative firm in a consumer goods sector along the equilibrium path. Since the goods markets clear in equilibrium, we can use (2.4) to solve for the inverted demand functions for consumer goods sectors as

$$\omega_t(Y_t) = \phi \left(W_t P_t^{\sigma-1}\right)^{1/\sigma} (Y_t)^{-1/\sigma} . \tag{2.17}$$

Intuitively, equilibrium prices for goods are ceteris paribus positively related to the aggregate index, holding fixed the consumption basket. Along an equilibrium path, for each t and state Ω_t, the optimality conditions for (H_t, I_t) are given, respectively, by

$$p_t F_H(K_t, H_t, A_t) = X_t, \tag{2.18}$$

$$\begin{aligned} Z_t + \upsilon\left(\frac{I_t}{K_t}\right) &= \mathbb{E}_t\Bigg[M_{t,t+1}\Bigg\{p_{t+1}F_K(K_{t+1}, H_{t+1}, A_{t+1}) + 0.5\upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \\ &\qquad (1-\delta)\left(Z_{t+1} + \upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\Bigg\}, \end{aligned} \tag{2.19}$$

[8] Conceptually, the extension of the equilibrium conditions to intermediate goods sectors in Proposition 1 below is straightforward, since it require substituting the consumer goods demand functions with intermediate goods demand functions from (2.12). However, extending the quantitative and empirical analysis to study the asset pricing implications of intermediate goods producers is an interesting avenue for future research.

where K_{t+1} is given by (2.10). The goods market clears in each sector so that

$$p_t = \phi \left(\frac{W_t}{P_t} \right)^{1/\sigma} P_t \left[F(K_t, H_t, A_t) \right]^{-1/\sigma} . \tag{2.20}$$

Finally, the equilibrium dividends D_t then satisfy (2.14), while the ex-dividend value S_t is given recursively by

$$\mathbb{E}_t \left[M_{t,t+1} \left(\frac{D_{t+1} + S_{t+1}}{S_t} \right) \right] = 1. \tag{2.21}$$

Equation (2.18) reflects the optimality condition that equates marginal cost of inputs (X_t) to the marginal revenue productivity of inputs at the competitive (or market clearing) price. Equation (2.19) is the Euler condition showing the trade-off between the current marginal cost of investment—represented by the left-hand side—with its discounted expected marginal value—given by or the right hand size.

3 Estimation of the Model

In this section, we undertake joint estimation of three salient parameters governing preferences, namely (γ, η, σ), through GMM. We focus on these parameters for estimation parsimony, which enhances (estimation) efficiency.[9] Furthermore, the joint estimation of risk aversion, IES and ES is novel and of intrinsic interest to both finance and applied economics literatures.

[9] In particular, when estimating the discount factor α, we do not obtain consistent or economically appealing estimates, even when estimating (γ, η, α), with exogenous values of ES. With some specifications, we obtain estimates exceeding 1, similar to other estimation attempts using additive utility (Hansen and Singleton (1983)) and recursive preferences (Epstein and Zin (1991)); or we obtain significantly negative values of the IES, which implies that consumption increases with interest rates.

3.1 Data

For empirical analysis of the model, we need industry data on capital, investment, materials input, sales, and productivity. We take these data from the NBER-CES manufacturing database. The latest data available are for 1958-2018 (annually). However, because industry productivity data are generally available only through 2016, our sample period is 1958-2016. The NBER-CES data are in nominal terms. While deflators for materials costs and investment are provided, the appropriate deflators for output and (especially) capital stock are not apparent. For this reason, we work with the nominal SDF in the Euler condition for investment (Equation 2.19) and, therefore, also asset returns. Consistent with the theoretical focus, we restrict attention to consumer goods industries subsample of the NBER-CES database by mapping 1997 North American Industry Classification System (NAICS) codes to four-digit 1987 Standard Industry Classification (SIC) codes.[10] Our industry variables are taken as the means of the corresponding variables in the data for consumer goods industries.

As the proxies for aggregate income (W) and consumption basket (C), we use per-capita national income and consumption expenditures from the Federal Reserve Bank of St. Louis (FRED).[11] We note that the aggregate price index in the model, P, is not necessarily the CPI, since along the equilibrium path consumption basket $C_t = \frac{W_t}{P_t}$. The appropriate price index is thus the "ideal" (or CES) price index, which is not generally available (see Redding and Weinstein (2020)). We therefore use the implied equilibrium P from the income and consumption data. We use the deflators for the material costs (H_t) and investment (I_t) as measures of X_t and Z_t.

We compute sectoral annualized monthly returns for as value-weighted portfolio returns of component industries. We take the sectoral annual returns as the value-weighted portfolio returns of all firms (in the sector). In particular, we first compute the value-weighted portfolio

[10] The list of consumer goods industry NAICS codes is obtained from Statistics Canada (2021).

[11] The FRED data are derived from the National Income and Product Accounts of the Bureau of Economic Affairs.

returns using monthly data and then compute the calendar year returns using the monthly time series. When needed, we apply the consumption price deflator (CPI) to adjust the returns data to real terms. The market and risk free returns are obtained from Kenneth French's website.

3.2 Basic Moment Conditions

Our model provides two "real side" moment conditions from the Euler conditions (2.18)-(EulerIC), as well as conditions from the asset market equilibrium (see (2.7) or (2.21)). These conditions form the basis of our estimation, and are specified succinctly below.

$$0 = \mathbb{E}_t \left[\phi \left(W_t\right)^{1/\sigma} \left(P_t\right)^{\frac{\sigma-1}{\sigma}} \left(Y_t\right)^{-\frac{1}{\sigma}} \left(\frac{Y_t}{H_t}\right) - X_t \right], \quad (3.1)$$

$$\begin{aligned} 0 &= -O_I(I_t, K_t) + \mathbb{E}_t \left[M_{t,t+1} \left\{ p_{t+1} F_K(K_{t+1}, H_{t+1}, A_{t+1}) - \right.\right. \\ &\left.\left. O_K(I_{t+1}, K_{t+1}) + (1-\delta) O_I(I_{t+1}, K_{t+1}) \right\} \right] = 0, \end{aligned} \quad (3.2)$$

$$0 = \mathbb{E}_t \left[M_{t,t+1} \tilde{\mathbf{R}}_{t+1} \right], \quad (3.3)$$

where (from Equation (2.13)) $O_I(I_t, K_t) = Z_t + \upsilon \left(\frac{I_t}{K_t}\right)$, $O_K(I_t, K_t) = -0.5\upsilon \left(\frac{I_t}{K_t}\right)^2$, and $\tilde{\mathbf{R}}_{t+1}$ is the vector of excess returns. In particular, we use the aggregate or market ERP, $\tilde{R}_t^{\Sigma} \equiv R_t^{\Sigma} - R_{ft}$, the consumer goods manufacturing sector (CM, j) ERP, $\tilde{R}_{jt} = R_{jt} - R_{ft}$, and the excess market returns relative to CM sector returns $\tilde{R}_t^{\Sigma,j} = R_t^{\Sigma} - R_{jt}$. In addition, and similar to Epstein and Zin (1991), we use the real aggregate return R_t^{Σ} as a proxy for R_{Ct}, that is, the gross return on the asset that pays aggregate consumption as its dividend.[12] In the usual way, we generate overidentifying restrictions through the use of instrumental variables (IVs), which we describe next.

[12]Hence, in our empirical tests, $M_{t,t+1} = P_t \left(\frac{\alpha^{\theta} \left(\frac{C_{t+1}}{C_t}\right)^{-\eta\theta} (R_t^{\Sigma})^{\theta-1}}{P_{t+1}} \right)$.

3.3 IVs and Time Series Characteristics

Because lagged endogenous variables are natural IVs, the sample own and cross-autocorrelations of endogenous variables in the moment conditions of our model are of particular interest. Panel A of Table 1 presents these correlations for the asset market conditions for one and two year lags. Consistent with the literature, there is relatively low cross-autocorrelations (for annualized observations) in per capita consumption growth G_t^C and aggregate ERP ($\tilde{R}_t^{\Sigma}$) in our sample. Not surprisingly, there is high contemporaneous correlation (0.75) between CM sector ERP ($\tilde{R}_{jt}$) and aggregate returns. But we also find that the own and cross-autocorrelation of $\tilde{R}_{jt}$ are essentially commensurate with those observed for aggregate returns. For example, the correlation between current and one-period lagged aggregate return is -0.08, while the corresponding correlation is about -0.06 for the industry returns. Furthermore, the cross-autocorrelation between lagged industry returns and current consumption growth is not significantly different than the corresponding correlations between market returns and consumption growth. In sum, utilizing industry returns in IV estimation would add information but not necessarily resolve the weak IV problem in structural estimation of asset pricing models.

In contrast, Panel B of Table 1 shows very high own and cross-autocorrelations in the industry-level industry investment (I_t) and materials input (H_t) in the CM sector. The high serial correlation in capital investment is also noted elsewhere in the literature (e.g., Eberly, Rebelo and Vincent (2012)). Thus, as we mentioned already, there is a potential here that industry investment and material inputs can be utilized as strong IVs in empirical estimation, to which we now turn.

3.4 Estimation with Equity Returns

To set up useful benchmarks, we first follow the standard approach and estimate (γ, η) by using the equilibrium asset return equation (3.3) and setting up moment conditions in terms of the aggregate (or market) equity risk premium (ERP), that is, $\mathbb{E}_t\left[\Gamma_t^{\Sigma}\left\{M_{t,t+1}\tilde{R}_{t+1}^{\Sigma}\right\}\right] = 0$. Here, $\mathbf{\Gamma}_t^{\Sigma}$ is a vector of IVs that—in the usual fashion—use only aggregate equity returns, the risk free return, and aggregate consumption data. Similar to the literature, we use lagged values of the aggregate risk premium as well as lagged consumption growth as IVs. In addition to the lagged covariates, we use an IV that is a nonlinear function of $(\tilde{R}_{t-1}^{\Sigma}, G_{t-1}^{C})$ to allow for nonlinear effects:

$$\Gamma_{nl,t}^{\Sigma} \equiv \left(\tilde{R}_{t-1}^{\Sigma}, G_{t-1}^{C}, \tilde{R}_{t-1}^{\Sigma} \times G_{t-1}^{C}, (\tilde{R}_{t-1}^{\Sigma})^2, (G_{t-1}^{C})^2\right). \tag{3.4}$$

Now, the SDF $M_{t,t+1}$ involves the discount factor, α. Because our production data are at an annual frequency, we use 3% annual discount rate, that is $\alpha = 0.97$, consistent with multisector general equilibrium models in the literature (e.g., Horvath (2000)).

The first two rows of Table 2 display the estimation results with the moment restrictions involving the market ERP, as well as tests of over-identifying restrictions through the chi-square statistic $\chi^2(DF)$. Based on the SDF for Epstein and Zin (1989) preferences, we estimate $\hat{\theta}$ and $\hat{\eta}$ and deduce the implied $\hat{\gamma}$ (see Equation (2.5)), which is displayed in parentheses next to $\hat{\theta}$. For various combinations of lagged covariates $(\tilde{R}_{t-\ell}^{\Sigma}, G_{t-\ell}^{C})$, $\ell = 1, 2, 3, 4$, we find statistically insignificant estimates with implausibly large values of risk aversion and very low (or even negative) values of IES that are also not economically appealing. For expositional convenience, the first row shows the results when we use the IV $(\tilde{R}_{t-\ell}^{\Sigma}, G_{t-\ell}^{C})$ with lags up to two years, and the second row displays the results when we utilize the nonlinear IV. The p-values of the J-statistics imply rejection of the overidentifying restrictions. The fragility of the risk aversion estimates, with a large change in the coefficient value across the two specifications, is indicative of the weak identification noted in the literature (Epstein and Zin (1991), Stock and Wright (2000)).

Because we have sectoral—that is, consumer manufactured goods—equity returns, we can undertake estimation using moment conditions in Equation (3.3) for industry returns, that is, utilize orthogonality conditions of the form $\mathbb{E}_t\left[\Gamma_t^j\left\{M_{t,t+1}\tilde{R}_{j,t+1}\right\}\right] = 0$. For this estimation, we also construct a nonlinear IV along the lines of (3.4), that is,

$$\Gamma_{nl,t}^j \equiv \left(\tilde{R}_{j,t-1}, G_{t-1}^C, \tilde{R}_{j,t-1} \times G_{t-1}^C, (\tilde{R}_{j,t-1})^2, (G_{t-1}^C)^2\right). \tag{3.5}$$

The results are displayed in the third and fourth rows of Table 2. In the third row, with lagged sectoral ERP and lagged consumption growth as IVs, we again get insignificant estimates and the estimates of risk aversion and IES are commensurate with the estimates in the second row. The fourth row displays the estimation results when we use the nonlinear sectoral IV, $\Gamma_{nl,t}^j$ (given in Equation (3.5)). But this exercise again yields insignificant estimates similar to those seen in the first three rows. Finally, we can combine aggregate and industry returns and use moment restrictions of the form $\mathbb{E}_t\left[\Gamma_t^{\Sigma,j}\left\{M_{t,t+1}\tilde{R}_{t+1}^{\Sigma,j}\right\}\right] = 0$ (where we recall $\tilde{R}_{t+1}^{\Sigma,j} \equiv R_{t+1}^{\Sigma} - R_{j,t+1}$). For the IVs here, we employ one year lagged market and sector returns. The results are again insignificant and the risk aversion estimate is substantially higher than earlier estimates and the IES estimate still remains very low.

In sum, estimating parameters of consumer preferences using only moment restrictions from the asset market equilibrium condition (3.3), utilizing both aggregate and consumer goods manufacturing industry returns with lagged returns and aggregate consumption growth as IVs, exhibits weak identification: The estimates are statistically insignificant and fragile, and yield implausible risk aversion and IES values. Our analysis is, therefore, consistent with the existing literature.

3.5 Estimation with Sectoral Production Data

We now use orthogonality conditions from the equilibrium path in both real (i.e., capital investment and materials) and financial variables to estimate the vector of unknown parameters (γ, η, σ). The real moment restrictions are given by Equations (3.1) and (3.2). With these two real restrictions, we potentially have 5 moment restrictions when we also include the three asset markets moment conditions used in the previous section, i.e., involving $\tilde{R}_t^{\Sigma}$, $\tilde{R}_{jt}$, and $\tilde{R}_t^{\Sigma,j}$.

However, there are well known pitfalls in adding moment conditions with fixed sample size, especially if they include weak moment conditions since increased estimation efficiency comes at the cost of increasing estimator bias (Han and Phillips (2006), Newey and Windmeijer (2009)). Furthermore, a large number of moment conditions raises the likelihood of mis-specification bias through utilization of possible invalid restrictions (Andrews (1999)). Consequently, our main test design for estimation uses only 4 moment restrictions: the two real moment conditions and two out of the three excess return restrictions involving market ERP or industry ERP or the excess aggregate return over the industry return. However, for robustness we also undertake estimation with all five moment conditions: two from the product market side and three types of excess return restrictions from the asset market side.

Because of the increased number of moment restrictions, we construct the IVs parsimoniously. Hence, we use only lagged returns for the asset market conditions, and lagged control variables—$H_{t-\ell}$ and $I_{t-\ell}$—for the corresponding optimality conditions ((3.1) and (3.2), respectively). Collectively, we denote these IVs as $\Pi_N, N = 1, 2, 3, 4$, and we will specify these when discussing the results.

As is apparent from Equations (3.1) and (3.2) (or from Equations (2.18)-(EulerIC)), utilizing the production and investment optimality conditions as moment restrictions requires specification of the sectoral utility weight ϕ; input production elasticities (ψ_K and ψ_H);

the depreciation rate δ; and the capital adjustment cost parameter υ. The estimated of percentage of income spent on manufactured consumer goods in the highest income decile countries (which includes U.S.) range from around 0.3 (Duarte and Restuccia 2016) to 0.39 (Duarte 2018). We thus set ϕ as 0.35 for our manufactured consumer goods sector. We estimate the production function (2.8) via GMM using our sectoral production data and estimate $\psi_H = 0.75$. Consistent with our competitive industries assumption, we set $\psi_K = 1 - \psi_H = 0.25$. We note that this capital factor share is close to the factor share of 0.3, which is often assumed in the real business cycle (RBC) literature.

Meanwhile, because of varying rates of depreciation for different types of capital (equipment, structures, and intellectual property), estimating depreciation rates is challenging. The literature notes that depreciation rates have been trending upwards because of the increased use of computer equipment and software since this lowers the useful life of capital stock (Oliner 1989). Moreover, the depreciation rates on such equipment have been rising. For example, Gomme and Rupert (2007) note that annual depreciation rates of computer equipment have risen from 15% in 1960-1980 to 40% in 1990s, and give estimates for software depreciation rates of about 50%. Epstein and Denny (1980) estimate the depreciation rate of physical capital (in the first part of our sample-period) to be about 13%. Because of increasing use of computerized and higher technology equipment in manufacturing during our sample period, we use an annual depreciation rate of 20%.[13] There is also a wide variation in the literature regarding estimates of the capital adjustment cost parameter υ. Utilizing US plant level data, Cooper and Haltiwanger (2006) find υ of around 10% for a strictly convex adjustment cost function; this is also consistent with small adjustment cost estimates in Hall (2004). We thus set $\upsilon = 0.1$.

The results of estimating (γ, η, σ) using real optimality conditions and sectoral production

[13] This value is also the mean depreciation rate estimated by a detailed study of Canadian manufacturing data (Gellatly et al 2007).

data are displayed in Table 3.[14] The first three rows are based on 4 moment conditions that include the aggregate and industry ERP restrictions, along with product market restrictions (3.1) and (3.2). The IV $\Pi_1 = \tilde{R}^{\Sigma}_{t-\ell}, \tilde{R}_{j,t-\ell}, \ell = 1,2,3; H_{t-\ell}, I_{t-\ell}, \ell = 1,2,3$. The IV Π_2 differs from Π_1 by setting $H_{t-\ell}, \ell = 1$, to examine the implications of asymmetry between short run utilization of material inputs versus the long run effects of investment on capital stock. The IV Π_3 differs from Π_2 by using only two year lags for the market risk premium, that is, $\tilde{R}^{\Sigma}_{t-\ell}$, $\ell = 1,2$. The specification in the fourth row, associated with Π_4, substitutes the moment restriction $\mathbb{E}_t\left[\Gamma^{j}_{-1}\left\{M_{t,t+1}(R^{\Sigma}_t - R_{jt})\right\}\right] = 0$ for the restriction $\mathbb{E}_t\left[\Gamma^{\Sigma}_{-2}\left\{M_{t,t+1}\tilde{R}^{\Sigma}_t\right\}\right] = 0$ and uses $R^{\Sigma}_{t-1}, R_{j,t-1}, I_{t-\ell}, \ell = 1,2,3$, and $H_{t-\ell}, \ell = 1$ as IVs. Finally, the specification Π_5 in the fifth row uses all 5 moment restrictions with the (equity return) IVs $\tilde{R}^{\Sigma}_{t-1}, \tilde{R}_{j,t-1}, R^{\Sigma}_{t-1}$ and $R_{j,t-1}$, along with the real IVs utilized in Π_2.

In striking contrast to the previous estimation results (Table 2), the point estimates of the unknown parameter vector (γ, η, σ) in each specification are statistically significant, with plausible values, and quite robust to changes in IVs and moment restrictions. The estimates of risk aversion (rounded to the nearest whole number) in all specifications are centered around 5. More specifically, four out of the five specifications in Table 3 generate risk aversion estimate of 5, while the other specification yields a risk aversion estimate around 6. Overall, the risk aversion estimates are well within the range of risk aversion values (2-10) considered plausible (or reasonable) by the literature (Mehra and Prescott (1985)). Comparing the statistically significant risk aversion estimates in Table 2 with those in Table 3, it is clear that utilizing equilibrium restrictions from industry-level real variables substantially reduces the risk aversion required to explain aggregate and CM sector equity risk premiums in the data.

Next, the IES estimates in Table 3 cluster around 0.2, and the empirical tests reject the null of zero IES at p-values significantly below 0.01. As we mentioned earlier, there is no

[14] For consistency of estimation procedures, the results in Table 3 use the same initial values (for θ and η) for GMM iterations as in Table 2.

consensus in the literature on the value of IES. With additive utility and using aggregate data, Hall (1988) finds low estimates of IES that are not significantly different from zero, and sets an upper bound of 0.1. Similarly, Epstein and Zin (1991) and van Binsbergen et al. (2008) estimate parameters of recursive preferences using different approaches and datasets but nevertheless report low estimates of the IES. More recently, using a novel source of quasi-experimental variation in interest rates (to address the challenge of finding exogenous variations in interest rates) and exploiting "notched" mortgage loan schedules in the UK that imply discrete jumps in mortgage rates at critical loan thresholds, Best et al. (2020) use individual home refinancing data to estimate IES around 0.1. More generally, a large number of studies using micro data report IES estimates below 0.4 (Havránek (2015), Havránek et al. (2015)).[15]

Note also that the estimates of $\hat{\theta}$ are reliably positive and strictly less than 1. This implies a negative relation of aggregate returns and the SDF, that is, declines in market returns generate increases in the SDF and hence the risk premium, which is empirically appealing. Indeed, this property is present in empirical parameterizations of Epstein and Zin (1989) preferences commonly used in the literature (e.g., Bansal and Yaron (2004), Croce (2014)), where the IES exceeds 1 (that is, $\frac{1}{\eta} > 1$) so that θ is negative (since risk aversion is generally taken to be higher than 1). However, the estimates in Table 3 imply a preference for late resolution of uncertainty because $\hat{\gamma}(1/\hat{\eta}) < 1$, which is consistent with other production and investment based asset pricing studies in the literature (e.g., Papanikolau (2011)).[16]

While we undertake our estimation with recursive preferences, where (relative) risk aversion and IES are treated as separate parameters, the majority of the specifications in Table

[15] However, the literature also reports estimates exceeding 1 (Vissing-Jorgensen (2002), Gruber (2013)).

[16] Evidence supporting the assumption of preference for early resolution of uncertainty is mixed. While some studies show that parameterization consistent with preference for early resolution can help match aggregate asset pricing moments (e.g., Bansal and Yaron (2004)), cross-sectional studies that examine the relation of risk-premia to investment maturities present confounding evidence: Binsbergen et al. (2012) find that claims to long-maturity dividends carry lower risk premia; and Giglio et al. (2015) and Weber (2018) find negative relation of risk premia and duration of risky cash flows.

3 imply an almost exact inverse relationship between these parameters.[17] As is well known, in the case of additive power expected utility, there is an exact inverse relationship between risk aversion and the IES. There exist other instances in the literature where estimates of risk aversion and IES are close to being in an inverse relationship (e.g., van Binsbergen et al. (2008)), although in different regions of the parameter space.[18] In a related vein, the estimation results imply that consumer risk preferences are not statistically different from a expected power utility specification, similar to Epstein and Zin (1991)—albeit with a different relative risk aversion coefficient.

Turning to the parameters relating to the CES product variety preferences, the estimates of the product elasticity of substitution (ES, σ) for the manufactured consumer goods sector cluster around 2. These estimates significantly exceed 1, which is the requirement of the theoretical model. The estimation of the CES in Dixit and Stiglitz (1977) preferences at the industry level is of long-standing interest in the applied economics literature. Broda and Weinstein (2006) use import data on product varieties from 1972-1988 and 1990-2001 and report median ES estimates for differentiated goods (at the four-digit Standard International Trade Classification (SITC) level) of 2.1 for 1990-2001 (and 2.5 for 19972-1988). Hottman and Monarch (2018) use import data from 1998 to 2014 and report the median ES for tradable consumer goods (at the four digit NAICS level) of 2.75. Thus, our estimates appear reasonably consistent with estimates of sectoral ES in the literature.

In sum, the results in Table 3 support the view that using a system of moment conditions and IVs—with both industry production and asset returns, along with market returns—lead to strong identification of the parameters related to consumer preferences in a general equilibrium model with product variety. This view is based on the uniform statistical significance of the point estimates for the entire parameter vector in Table 3, their low dispersion (compared with Table 2 for (γ, η)), and economic appeal.

[17] In the case of specification Π_3, the risk aversion is strictly greater than the inverse of the IES.

[18] van Binsbergen et al. (2008) estimate the IES as 0.06 and risk aversion as 46 (whose inverse is 0.02).

The strong identification of consumer preference parameters with economically reasonable values in Table 3 is in striking contrast to the estimation results in Table 2. Of course, there are several differences in the empirical tests underlying the results in Table 2 and Table 3. These include the expansion of preference parameters to include the ES, utilization of industry-level (real) Euler conditions for investment and production, and use of industry production data in the IVs. It is therefore of substantial interest to examine the relative contributions of these differences for the estimation performance in Table 3. Moreover, the estimates in Table 3 suggest that the model (in Section 2) is consistent with the industry-level equity returns in the data with reasonable values of risk aversion, IES, and ES. But it would also be useful to verify this quantitatively. Our analysis in the next two sections pursues these issues.

4 Euler Conditions, IVs and Estimation Efficiency

The estimation in Table 3 differs from the one using only asset returns—displayed in Table 2— in three ways. First, we explicitly incorporate consumer preferences over product variety through the ES, σ, in the estimation. Second, we add industry-level real moment conditions (3.1) and (3.2). Third, we use industry production data in the IVs, which helps exploit their stronger own- and cross-autocorrelations as seen in Table 1. It is useful to examine the effects of these three changes in enhancing the identification in Table 3 (relative to Table 2).

4.1 Effects of product variety and industry Euler conditions

We first examine whether the improved estimation of risk aversion and IES in Table 3 is an artefact of raising the number of estimated parameters from 2, i.e., (γ, η), to 3, i.e., (γ, η, σ). The alternative hypothesis is that incorporating preferences over product variety (ES) in the canonical asset pricing model adds structural restrictions—i.e., derived from the equilibrium conditions of our model—that enhance identification of all the parameters of interest, in particular, (γ, η). To disentangle these two hypotheses, we undertake estimation by using the moment conditions and IVs in Π_1, but fixing the value of $\sigma = 2.03$, consistent with the estimate in Table 3. As seen in the first row of Table 4, we find reliable estimates of (γ, η) essentially identical to that in Table 3 (for Π_1). We conclude that it is the incorporation of preferences over product variety, rather than the higher number of estimated parameters per se, that improves identification of risk aversion and IES.

Turning to the role of the moment conditions derived from the industry equilibrium, the improved identification or estimation efficiency in Table 3 could arise from the use of one or both of the Euler conditions. Hence, it is useful to examine the relative roles of the intertemporal Euler condition for investment and the intratemporal materials input Euler condition. To facilitate comparison with Table 3, we maintain the IVs for moment conditions on returns given in Π_1. (We label the specifications with "bars" to reflect constrained estimation.) We first use only the investment Euler condition along with the asset returns moment conditions and IVs in Π_1. We are unable to reliably estimate all three parameters (γ, η, σ). However, using $\sigma = 2.03$ and estimating only (γ, η) generates estimates similar to Table 3, as seen in the second row of Table 4. Comparing these estimates with Table 2 (where we used only the asset returns moment condition), we conclude that adding the intertemporal investment Euler equation significantly enhances identification of risk aversion and IES *conditional* on knowledge of σ.

Next, the third row of Table 4 presents estimation results when we include the material optimality condition but exclude the investment Euler condition. In this case, we obtain a reliable estimate of ES, which is consistent with the estimation in Table 3. However, the risk aversion and IES estimates are highly distorted relative to the estimates in Table 3—the former being too high and the latter being too low, and there is also a significant decline in estimation precision of θ. Hence, we conclude that estimation efficiency and/or the economic appeal of estimates appear to significantly deteriorate if either of the two product market conditions—the investment Euler condition and the materials input optimality condition—are excluded.

The fourth row of Table 4 shows estimates from utilizing the materials input and investment optimality conditions, while excluding the asset returns moment condition. The estimates are significant and close to the estimates in Table 3. However, the risk aversion estimate is lower than the robust estimate in Table 3. These results are consistent with the view that the production data—that is, capital investment and materials input data—serve as strong IVs and significantly enhance identification of parameters governing consumer preferences.

In sum, explicitly modeling consumer preferences over product variety and utilizing both the intertemporal and intratemporal industry production Euler conditions—as well as the strong industry-based IVs—are together required for enhancing the estimation precision and economic appeal of parameters governing consumer preferences in a generalized setting with product variety. We also show that adding lagged industry and market returns as IVs further improves estimation efficiency. Our results complement findings in other areas of finance where richer parameterization of consumer preferences and using novel data improves identification. For example, Huang and Shaliastovich (2014) show that using recursive preferences—in particular, imposing parameteric restrictions for early resolution of uncertainty—and using equity options data helps identification of objective probabilities and risk adjustments.

4.2 Effects of industry production data in IVs

We examine next the effects using industry production data in IVs. To do so, we replace the production-data-based IVs in Table 3 with aggregate IVs utilizing aggregate per capita consumption growth. That is, in place of industry-level investment and materials expenditures data that we utilized in IVs in the estimation in Table 3, we use consumption growth. The lag structure for consumption growth in the IVs is the same as the lag structure for industry data in the original (Table 3) IVs. We label the new IVs as $\Pi'_j, j = 1, ...5$, for convenience.

The results are displayed in Table 5. Comparing the results in Table 5 with those in Table 3, we find that the estimates for risk aversion and IES are no longer statistically significant. While the risk aversion levels are reasonable, the estimates for IES are not appealing, essentially being 0. On the other hand, the estimates of ES, σ, are essentially the same as in Table 2 and statistically significant. We also note that the p-value for the J statistics are larger for each specification in Table 5 compared with the corresponding specification in Table 3. Overall, we conclude that IVs using industry production data significantly improve estimation efficiency relative to IVs using aggregate consumption data.

The foregoing analysis suggests that incorporating consumer preferences over product variety—parameterized in our model by the ES (σ)—appears to add structural restrictions to the canonical asset pricing model, which helps explain the strong identification of risk aversion and IES (along with the ES) seen in Table 3. More precisely, if the ES significantly affects equilibrium risk premium, then making it explicit in the equilibrium conditions (2.18)-(2.21) imposes additional structural restrictions on equilibrium returns relative to models where the role of the ES is suppressed. It is, thus, important to examine whether and how the ES affects equilibrium risk premium. In particular, the moderate risk aversion and low IES estimates in Section 3 suggest that our model can generate sizeable industry-level risk premiums around the (γ, η, σ) estimates in Table 3, and it would be useful to verify this quantitatively.

More generally, the relation of consumer preferences for product variety and equity risk premiums is of independent interest. In the next section, we quantitatively examine relation of the product elasticity of substitution (i.e., ES) and the equity risk premium.

5 Preferences for Product Variety and Risk Premium

There is an intuition that industries whose products are characterized by *low* elasticity of substitution with alternative goods—for example, luxury or high product differentiation goods—will have profits that are more cyclically sensitive relative to industries with high elasticity of substitution, such as commodities. Because greater procyclicality in profits earns higher risk premium, other things being equal, it follows that there should be a negative relation of elasticity of substitution (ES) and ERP. In this section, we utilize our model to quantitatively assess this hypothesis. In addition, we explore the interaction of ES with risk aversion, capital productivity, and adjustment costs in terms of the effects on ERP.

5.1 Analytic Approximations

Our analysis proceeds through loglinear approximations around product and asset market equilibrium of our model in deterministic steady state (King et al. (1988), Uhlig (1995)). We apply this approach *industry* equilibria, along the lines specified in Equations (2.18)-(2.21), where the aggregate income and price index (W_t, P_t) as well as the (industry) productivity and input price indices (A_t, X_t, Z_t) are taken as state variables.

The details of the analytic approximations, including the industry and asset market equilibria in the steady state, are given in the Online Appendix. Because the only exogenous shocks to our economy, namely, the sectoral productivity shocks follow a first order log-autoregressive process (see Equation (2.9)), we will solve the model with loglinear approximations of industry and asset markets' equilibrium under the assumption that the industry stochastic state variables $(W_t, P_t, A_t, X_t, Z_t)$ follow a log-autoregressive system with multivariate normal i.i.d correlated shocks. The model's solution will then give, at any t, the sector capital stock K_{t+1}, product prices p_t, the log of dividends d_t, as well as the log of the SDF m_t and log of equity prices s_t as linear combinations of the industry's state variables.

Specifically, using the notation: $w_t \equiv \log(W_t), \pi_t \equiv \log(P_t)$, $a_t = \log(A_t)$, $x_t = \log(X_t)$, $z_t = \log(Z_t)$, we let $\boldsymbol{\mu}_t \equiv (w_t, \pi_t, a_t, x_t, z_t)'$, $\boldsymbol{\rho} = (\rho_w, \rho_\pi, \rho_a, \rho_x, \rho_z)$, $0 \leq \boldsymbol{\rho} \leq 1$, and $\boldsymbol{\varepsilon}_t = (\varepsilon_{wt,}\varepsilon_{\pi t,}\varepsilon_{at,}, \varepsilon_{xt,}\varepsilon_{zt})'$. Then the log-state vector $\boldsymbol{\mu}_t$ follows the recursive law of motion

$$\boldsymbol{\mu}_{t+1} = \boldsymbol{\rho}'\boldsymbol{\mu}_t + \boldsymbol{\varepsilon}_{t+1}. \tag{5.1}$$

Now, $\boldsymbol{\mu}_t$ includes two aggregate quantities—w_t and π_t—that are common across industries—and three industry-specific quantities: a_t, x_t and z_t. Correspondingly, the shock vector $\boldsymbol{\varepsilon}_t$ is also composed of aggregate and industry-specific shocks. In particular, $\boldsymbol{\varepsilon}_t$ are multivariate normal mean zero variables with the variance-covariance matrix $\Phi = \left[\Phi_{ij}\right]$, where $Cov(\varepsilon_{wt,}\varepsilon_{wt,}) = \Phi_w^2, Cov(\varepsilon_{\pi t,}\varepsilon_{\pi t,}) = \Phi_\pi^2$ and $Cov(\varepsilon_{wt,}\varepsilon_{\pi t,}) = \Phi_{w\pi}$ are common across , while $Cov(\varepsilon_{wt,}\varepsilon_{at}) = \Phi_{aw}$ etc. are specific to the industry. Under the maintained assumption of a stationary variance-covariance matrix of the aggregate and industry shocks, it is shown in the Online appendix that the unconditional industry ERP can be written as:

$$\mathbb{E}[r_{t+1} - r_{t+1}^f] = \beta_w\Phi_w^2 + \beta_\pi\Phi_\pi^2 + \beta_{w\pi}\Phi_{w\pi} + \beta_{aw}\Phi_{aw} + \beta_{xw}\Phi_{xw} + \beta_{zw}\Phi_{zw} + \beta_{a\pi}\Phi_{a\pi} + \beta_{x\pi}\Phi_{x\pi} + \beta_{z\pi}\Phi_{z\pi}. \tag{5.2}$$

In (5.2), β_w, β_w and $\beta_{w\pi}$ represent the risk loadings of the representative firm in sector j to aggregate shocks, that is, exposure to income and price index shocks. And $\beta_{aw}, \beta_{xw}, \beta_{x\pi}, \beta_{zw}$, and $\beta_{z\pi}$ represent the industry's risk sensitivities to industry-specific shocks due to the covariation between shocks to industry productivity and intermediate cost indices with aggregate shocks. The specification of the loadings, in terms of the parameters of the equilibrium (log) dividend and stock price functions are provided in the Online appendix.

The ERP representation in (5.2) differs from the received literature in two principal ways. First, unlike single consumption good macrofinance models—both in exchange and production economies—our model allows cross-sectional variation across industries. Second, and distinct from the production-based asset pricing literature, the "real side" of our model specifies the contribution to equilibrium ERP due to the covariation between industry-specific (productivity and materials costs) shocks and aggregate shocks. Furthermore, because of their independent effects on product demand, the aggregate consumption basket is not a sufficient statistic for aggregate income and price index with endogenous production. To explicate, note that consumer optimum with CES preferences implies that $C_t = \frac{W_t}{P_t}$, where C_t is the aggregate consumption basket, W_t is the aggregate income and P_t is the (CES) aggregate price index.[19] In particular, with output Y_t, the inverse demand function $\omega_t(Y_t)$ is proportional to $(W_t/P_t)^{1/\sigma} P_t (Y_t)^{-1/\sigma}$, where σ is the ES. Hence, the industry demand curve shifts outward with exogenous increases in aggregate income *or* the price index—that is, C_t does not subsume the effects of the aggregate risk factors on endogenous production—so that W_t and P_t operate as separate aggregate risk factors. Hence, the risk premium is affected by volatilities of aggregate income and price index, as well as their covariation.

Finally, the risk premium is also affected by the covariation between sectoral productivity shocks and intermediate goods prices with the aggregate income and price index. For example, a positive covariation between sectoral *input* prices and the aggregate price index (P_t) has a positive risk premium because higher production and investment costs—and hence lower cash flows—tend to coincide with low higher real consumption $\frac{W_t}{P_t}$ and higher stochastic discount factor (SDF). With appropriate calibration, (5.2) allows the computation of the unconditional ERP. We now turn to these computations.

5.2 Calibration

The asset pricing literature highlights the role of durability in expected returns (Yogo (2008), Gomes, Kogan and Yogo (2009)). We therefore focus on the consumer durable (manufac-

[19] Consistent with the CES model, we derive the CES price index as $P_t = W_t/C_t$, using NIPA income and consumption data. Hence, by construction, the variance and covariance of log changes in aggregate income and price index utilized in our analysis imply the (annual) volatility of log per capita consumption growth in the data. In sum, our results are not driven by "large" values of implied consumption growth volatility.

tured) goods sector to calibrate our quantitative analysis, which requires parameterization for aggregate quantities (which are common across sectors), as well as the sector-specific parameters. As in our estimation analysis in Section 3, we use per-capita national income and consumption expenditures from FRED.

The (annual) volatility of per capita national income shocks (Φ_w) in our sample period is 2.47% , while the volatility of the shocks to log aggregate price index (Φ_π) is very low, that is, 0.76%; and the covariance $\Phi_{w\pi}$ is even lower (4.8 e-05). Using the relation $\log C_t = \log W_t - \log P_t$, we obtain (annual) volatility of log consumption growth as 2.39%. By construction (since P is forced to equal W/C), this volatility equals (up to rounding) the annual volatility of per-capita consumption in our sample period. We also derive the persistence parameters (in levels) for the aggregate variables, i.e., (ρ_w, ρ_π), from the data. The sector-specific parameterization is based on the NBER-CES data. In particular, we calibrate from the production data the covariances between the industry and aggregate factors $(\Phi_{\cdot w}, \Phi_{\cdot \pi})$ and the persistence parameters for industry productivity, materials deflator, and investment deflator, (ρ_a, ρ_x, ρ_z). This calibration is specified in Table 5. The covariance between aggregate and industry factors are very small. And the high persistence in the macroeconomic variables is consistent with the RBC literature ((Prescott (1986), Milani (2007)), while the persistence of industry productivity is consistent with the asset pricing literature (Kaltenbrunner and Lochstoer (2010), Papanikolaou (2011)).

We use other model parameterization consistent with our empirical estimation in Section 3. Specifically, $\alpha = 0.97$, $\phi = 0.35$, $\delta = 0.2$, and $\upsilon = 0.1$. The parameterization of production elasticities (ψ_K and ψ_H) for the consumer durable goods sector follows a procedure similar to that described in Section 3. We then analyze the relation of consumer preference parameters (γ, η, σ) and the ERP around our structural estimates from our model using GMM (see Section 3). Specifically, as the baseline parameterization, we take risk aversion (γ) to be 5; the IES (η^{-1}) as 0.19; and the intratemporal ES (σ) as 2.03. Following Feenstra (1994) and

Broda and Weinstein (2006), the interpretation of this sector-specific ES in our setting is the elasticity of substitution between durable and nondurable consumer manufactured goods.[20] Table 5 also records the values of these parameters for convenience.

In the usual fashion, we take the steady state value of per capita income $\bar{W}$ and aggregate (CES) price index $\bar{P}$ as their sample means. By construction of the P_t time-series, the implied steady state consumption-to-income ratio matches the sample mean and the steady-state consumption level is also close to its sample mean. Similarly, the steady state values $\bar{A}$, $\bar{X}$, and $\bar{Z}$ are calibrated so that the model's steady state per capita output, materials inputs, and capital stock match their corresponding sample means. The procedure for computing the long-linearized SDF through simulations is given in the Online appendix (Section C.1).

5.3 Asset Pricing Implications

Figure 1 displays surface plots of the effects on ERP of bivariate variations in the ES (σ) and risk aversion (γ). The figure confirms that, for a fixed σ, ERP is positively related to risk aversion. But the ERP is negatively related to the ES. In the CES setting (see (2.4)), the sectoral ES measures the demand price elasticity of the industry good; hence, the graphical analysis in Figure 1 indicates a negative relation of ERP and price elasticity.

Conceptually and empirically, goods with low product differentiation, such as commodities and basic consumption goods, have greater demand price elasticity compared with highly differentiated goods, such as luxury goods (Berry, Levinsohn and Pakes (1995), Broda and Weinstein (2006)). As we mentioned already, high price-cost markup differentiated goods are exposed to much greater cyclical risk compared to basic consumption goods. Risk averse

[20] This approach is based on extending the basic CES basket (2.4) specification to allow for nonsymmetric or good-specific ES (e.g., Broda and Weinstein (2006)). Using this more flexible specification does not change the theoretical characterization of the equilibrium and, hence, the sectoral asset pricing implications. For notational parsimony, we continue to use σ to denote the sectoral ES in the manufactured consumer durable goods sector.

investors should therefore demand higher compensation for being exposed to the risk of greater (negative) covariance of the SDF with the returns of differentiated goods industries, compared with low differentiation industries. The graphical analysis in Figure 1 is consistent with this intuition.

Figure 1 also shows interaction effects of risk aversion and ES on the ERP, which are in line with the intuition given above. In particular, the negative relation of ERP and ES increases in magnitude with risk aversion. For example, the ERP gradient with respect to σ (from $\sigma = 3$ to 1.5) is 1.3 when $\gamma = 5$, while this gradient is 2.6 when the $\gamma = 10$. Because of the interaction effects of σ and γ, ERP can be sizeable for γ around 10 and σ around 1.5 (around 4.67%); and[21] conversely the ERP can be relatively small for risk aversion of 10 for high demand elasticity goods (around 2.05%).[22]

Because of the prominent role of the IES in asset pricing with Epstein and Zin (1989) preferences, it is also of interest to examine the interaction of the intratemporal ES between products and the IES (η^{-1}). This graphical analysis is presented in Figure 2. ERP is negatively related to IES (for a given σ) and, of course, we see the negative relation of σ and ERP (for a given IES). Moreover, the negative effect of ES and ERP increases in magnitude as IES falls. For example, the ERP gradient with respect to σ (from $\sigma = 3$ to 1.5) is 1.25 when IES is 0.4 ($\eta = 2.5$), while this gradient is 1.4 when the IES is 0.1 ($\eta = 10$). Overall, the effects of IES are opposite of the effects of risk aversion seen in Figure 1. There, ERP is positively related to risk aversion and the negative effect of ES on ERP is increases with risk aversion. This is consistent with the estimation results in Section 3 where we find an inverse relation of risk aversion and IES.

It is also apparent that our general equilibrium model with multiple consumption goods (or product variety), capital investment and production generates sizeable equity risk-premiums

[21] As a benchmark, the annual ERP for the consumer durable sector in our sample period is 6.71%.

[22] As a benchmark, the mean annualized equity risk-premium for the durable consumption goods manufacturing during our sample period is 6.71%.

with reasonable calibration of parameters governing consumption preferences, as well as production-related parameters consistent with the data and/or the literature. This analysis is thus consistent with the estimates of consumer preferences parameters in Table 3. The sizeable equity risk-premiums generated by our model for reasonable parameterization complement the existing literature on multisector models with investment and production in the presence of capital adjustment costs (e.g., Papanikolaou (2011)).

Finally,for simplicity, we do not incorporate non-dividend income for the CI in the model. An exogenous stochastic non-dividend income component will not significantly affect the results since the analytic approximations above are based on a general first order log-autoregressive process for consumer wealth and we calibrate the model with per capita income in the data. Allowing endogenous labor income with leisure as a component of consumer preferences and labor as a factor of production will not significantly affect the endogenous risk factors, but will affect the factor loadings.

6 Summary and Conclusions

Estimating preference parameters of the representative agent using GMM and Euler conditions from asset pricing models—such as the CCAPM—is of long-standing interest in financial economics. The received literature typically uses lagged consumption and (aggregate) returns as instrumental variables (IVs). But these IVs are weak because of low autocorrelation in returns as well as low cross-autocorrelations in consumption and returns; hence, these tests typically suffer from a weak identification problem. In this paper, we present a novel estimation approach building on the view that value-maximization by firms in multisector settings implies that consumer preference parameters are reflected in industry-level production information. Developing a dynamic multisectoral production-based general equilibrium asset pricing model where the representative consumer is endowed with recursive CES preferences on baskets of goods and firms choose material inputs and capital investment, we use real restrictions from and U.S. manufacturing data for GMM estimation of risk aversion, IES, and ES. Industry investment and production data provide strong instruments due to high autocorrelations. We obtain economically appealing, efficient estimates of risk-aversion, IES, and ES. In contrast, we find weak identification when we utilize only standard asset market restrictions and aggregate returns and consumption.

Our analysis is consistent with the view that incorporating preferences over product variety (parameterized by the ES) in the canonical asset pricing model adds structural restrictions that enhance identification of all the parameters of interest, in particular, risk aversion and IES. Furthermore, tests of subsets of moment conditions of the model indicate that Euler conditions from the industry production equilibrium and industry IVs are critical for identification. In particular, the intertemporal investment Euler condition contributes significantly to the identification of risk aversion and IES, whereas the intratemporal materials input condition contributes significantly to the identification of ES. Consequently, both the real restrictions contribute to the identification of risk aversion, IES, and ES.

The structural restriction added by considering preferences over product variety is due to the effect of ES on the risk premium. There is a strong economic intuition that, at the industry level, ERP should ceteris paribus be positively related to product differentiation. This is because commodities and basic consumption goods, with low product differentiation, exhibit small cyclical variation in price-cost markups and, hence, profits. In contrast, highly differentiated goods have relatively large cyclical variation in profit margins, amplifying the negative covariation between the stochastic discount factor and returns. Because product demand elasticity is positively related to the ES at consumer optimum, this argument predicts a negative relation of ES and ERP. Our quantitative analysis verifies that ES and ERP are negatively related.

The framework developed in this paper can be extended in several directions in future research. We have focused on the asset pricing implications and estimation of consumer goods sectors. A natural extension of our study is to examine asset pricing in intermediate goods industries. Furthermore, for analytic tractability, we have not considered endogenous labor income in this study; a more realistic model with endogenous labor input in production, with attendant labor income for consumers, would be a natural extension in future research. The production environment can also be enhanced by allowing innovation (or growth) in industry productivity and allowing entry and exit.

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Table 1. Matrix of Autocorrelation Coefficients

Variables	G_t^C	G_{t-1}^C	G_{t-2}^C	$\tilde{R}_t^\Sigma$	$\tilde{R}_{t-1}^\Sigma$	$\tilde{R}_{t-2}^\Sigma$	$\tilde{R}_t^j$	$\tilde{R}_{t-1}^j$	$\tilde{R}_{t-2}^j$
G_t^C	1.000								
G_{t-1}^C	0.910	1.000							
G_{t-2}^C	0.780	0.905	1.000						
$\tilde{R}_t^\Sigma$	0.104	-0.072	-0.025	1.000					
$\tilde{R}_{t-1}^\Sigma$	0.115	0.121	-0.062	-0.078	1.000				
$\tilde{R}_{t-2}^\Sigma$	-0.084	0.100	0.108	-0.228	-0.077	1.000			
$\tilde{R}_t^j$	0.185	-0.152	0.030	0.751	-0.216	-0.415	1.000		
$\tilde{R}_{t-1}^j$	0.217	0.217	0.011	-0.135	0.755	-0.208	-0.057	1.000	
$\tilde{R}_{t-2}^j$	-0.002	0.194	0.191	-0.205	-0.134	0.756	-0.338	-0.034	1.000

B: Industry Production Variables

Variables	I_t	I_{t-1}	I_{t-2}	H_t	H_{t-1}	H_{t-2}
I_t	1.000					
I_{t-1}	0.971	1.000				
I_{t-2}	0.934	0.972	1.000			
H_t	0.972	0.955	0.941	1.000		
H_{t-1}	0.964	0.973	0.984	0.990	1.000	
H_{t-2}	0.943	0.966	0.973	0.979	0.990	1.000

Notes to Table: This table uses annual data from 1958 to 2016. Panel A presents own and cross-autocorrelations of yearly growth rates of U.S. per capita consumption growth G_t^C, annualized aggregate (market) ERP $\tilde{R}_t^\Sigma = R_t^\Sigma - R_{ft}$, where R_t^Σ is the value-weighted CRSP return and R_{ft} is the annual risk free rate, and annualized ERP $\tilde{R}_{jt}$ from value-weighted portfolio of consumer goods manufacturing industries in the NBER-CES database. Panel B presents own and cross-autocorrelations of mean annual industry investment (I), materials input (H), and average productivity (A) using the NBER-CES manufacturing database.

Table 2. Estimation using only asset market restrictions

IV	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	J	DF	p-Value
Γ^{Σ}_{-2}	-0.002 (68)	1.730	0.00	2.890e+07	1.07	1	0.30
Γ^{Σ}_{nl}	-0.49 (56)	0.783	- 0.01	196.272	0.85	4	0.93
Γ^{j}_{-2}	-0.006 (51)	1.624	0.00	2.206e+6	1.31	3	0.73
Γ^{j}_{nl}	-0.662 (47)	0.914	-0.01	106.019	1.69	4	0.79
$\Gamma^{\Sigma,j}_{-1}$	2.00 (113)	1.12	0.02	56.298	0.015	1	0.90

Notes to Table: This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$) and the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$) from moment restrictions derived from the asset market equilibrium condition, using data on aggregate ($\tilde{R}^{\Sigma}_t$) and manufacturing industry ($\tilde{R}_{jt}$) equity risk premium. The sample period is 1958-2016 (annual) and the data are described in the text. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

Table 3. Estimation using production and asset market restrictions

IV	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	χ^2	DF	p-Value
Π_1	0.879 (5)***	0.150	0.19***	0.626	2.03***	0.022	8.155	13	0.833
Π_2	0.865 (5)***	0.185	0.19***	0.992	2.02***	0.023	6.049	11	0.870
Π_3	0.864 (6)***	0.188	0.19***	1.1540	2.02***	0.023	6.048	10	0.811
Π_4	0.861 (5)***	0.190	0.19***	1.059	2.02***	0.030	5.979	10	0.817
Π_5	0.871 (5)***	0.182	0.19***	0.922	2.02***	0.021	6.074	12	0.912

Notes to Table: This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$), the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$), and the intratemporal elasticity of substitution ($\hat{\sigma}$) from data on U.S. consumer goods manufacturing industries. The sample period is 1958-2016 (annual) and data sources are described in text. The moment restrictions are derived from the Euler conditions for capital investment and materials input, as well as the asset markets considered in Table 2. The other parameters used in the moment conditions are set as $\alpha = 0.97$, $\phi = 0.35$, $\psi_H = 0.75$, $\psi_K = 0.25$, $\upsilon = 0.1$, and $\delta = 0.2$. The sample period is 1958-2016 (annual) and data sources are described in text. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

Table 4. Role of production restrictions

Euler Equations	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	χ^2	DF	p-Value
All, $\sigma = 2.03$	0.888 (5)***	0.129	0.19***	0.515			5.707	14	0.973
Investment, Asset Markets, $\sigma = 2.03$	0.847 (5)***	0.222	0.19***	1.5442			7.284	10	0.698
Material Inputs, Asset Markets	5.817 (836.2)*	3.049	0.01***	29.817	2.04***	0.044	2.892	3	0.409
Investment, Material Inputs	0.887 (4)*	0.146	0.23**	0.529	2.05***	0.043	5.482	5	0.360

Notes to Table: This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$), the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$), and the intratemporal elasticity of substitution ($\hat{\sigma}$) from data on U.S. consumer goods manufacturing industries. The sample period is 1958-2016 (annual) and data sources are described in text. The combinations of moment restrictions are derived from the Euler conditions for capital investment and materials inputs, and asset markets, as considered in Table 3. We use the instrumental variables specified in Π_1 in Table 3. The other parameters used in the moment conditions are set as $\alpha = 0.97$, $\phi = 0.35$, $\psi_H = 0.75$, $\psi_K = 0.25$, $\upsilon = 0.1$, and $\delta = 0.2$. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

Table 5. Role of production IVs

IV	$\hat{\theta}(\hat{\gamma})$	SE($\hat{\theta}$)	$\hat{\eta}^{-1}$	SE($\hat{\eta}$)	$\hat{\sigma}$	SE($\hat{\sigma}$)	χ^2	DF	p-Value
Π_1'	0.003 (4)	0.189	0.001	5.949e+4	1.97***	0.022	4.506	13	0.984
Π_2'	0.001 (3)	0.305	0.00	5.064e+5	1.96***	0.0214	4.473	10	0.924
Π_3'	0.001 (3)	0.204	0.00	2.225e+5	1.97***	0.022	4.408	10	0.927
Π_4'	0.002 (3)	0.182	0.001	1.109e+5	1.97***	0.020	4.739	10	0.907
Π_5'	0.003 (3)	0.188	0.001	4.257e+4	1.96***	0.013	4.566	12	0.971

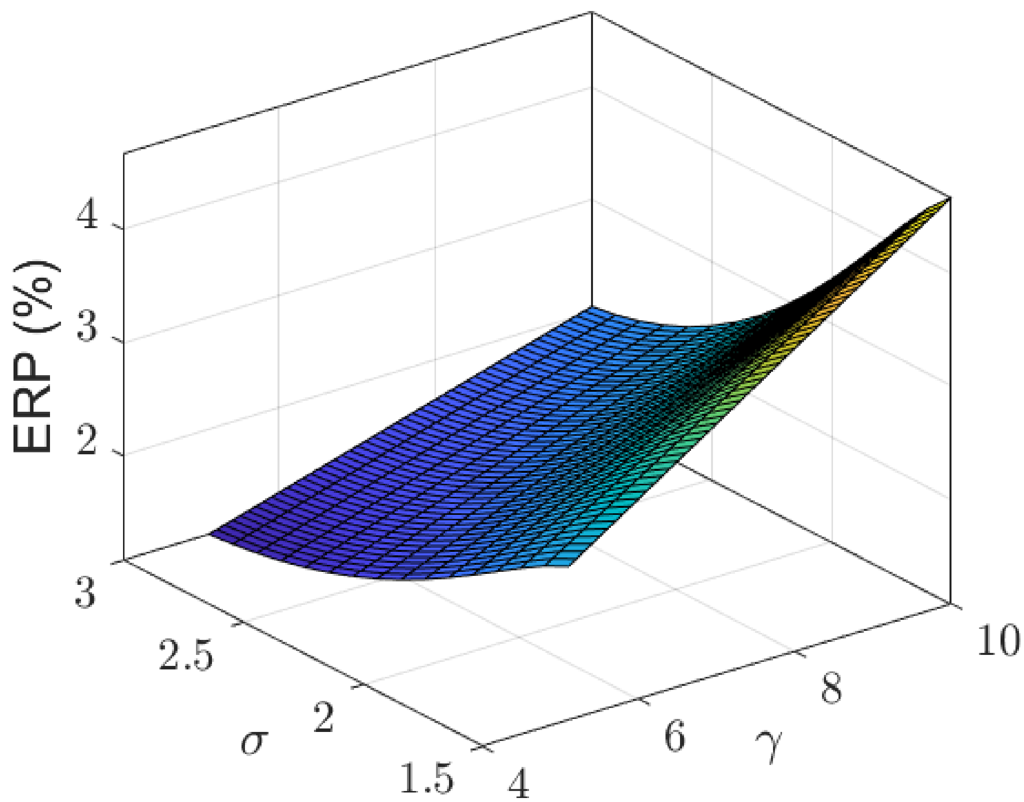
Notes to Table: This table presents the point estimates, standard errors, and J statistics from two step GMM estimation (with heteroskedasticity- and autocorrelation-consistent inference) of risk aversion ($\hat{\gamma}$), the intertemporal elasticity of substitution of consumption ($\hat{\eta}^{-1}$), and the intratemporal elasticity of substitution ($\hat{\sigma}$) from data on U.S. consumer goods manufacturing industries. The sample period is 1958-2016 (annual) and data sources are described in text. The combinations of moment restrictions are derived from the Euler conditions for capital investment and materials inputs, and asset markets, as considered in Table 3. To construct instrumental variables in Table 5, we start with instrumental variables $\Pi_1 - \Pi_5$ in Table 3 and replace industry production variables with aggregate consumption growth resulting in ($\Pi_1' - \Pi_5'$). The other parameters used in the moment conditions are set as $\alpha = 0.97$, $\phi = 0.35$, $\psi_H = 0.75$, $\psi_K = 0.25$, $\upsilon = 0.1$, and $\delta = 0.2$. The p-value of J statistics are calculated with Chi-square distribution with degrees of freedom DF. Statistical significance at 10%, 5%, and 1% levels are denoted by *,** , and ***, respectively.

Table 6. Calibration for quantitative analysis

	Aggregate Parameters							Sectoral Parameters			
Parameter	Φ_w	Φ_π	$\Phi_{w\pi}$	ρ_w	ρ_π	α	γ	η^{-1}	υ	δ	σ
	2.47%	0.76%	4.8e-05	0.95	0.965	0.97	5	0.19	0.1	0.2	2.03
	Sectoral Parameters										
Parameter	ψ_K	ψ_H	Φ_{aw}	$\Phi_{a\pi}$	Φ_{xw}	$\Phi_{x\pi}$	Φ_{zw}	$\Phi_{z\pi}$	ρ_a	ρ_x	ρ_z
	0.21	0.79	4e-05	4.2e-05	3.9e-04	5.5e-05	3e-04	-1.2e-07	0.930	0.966	0.962

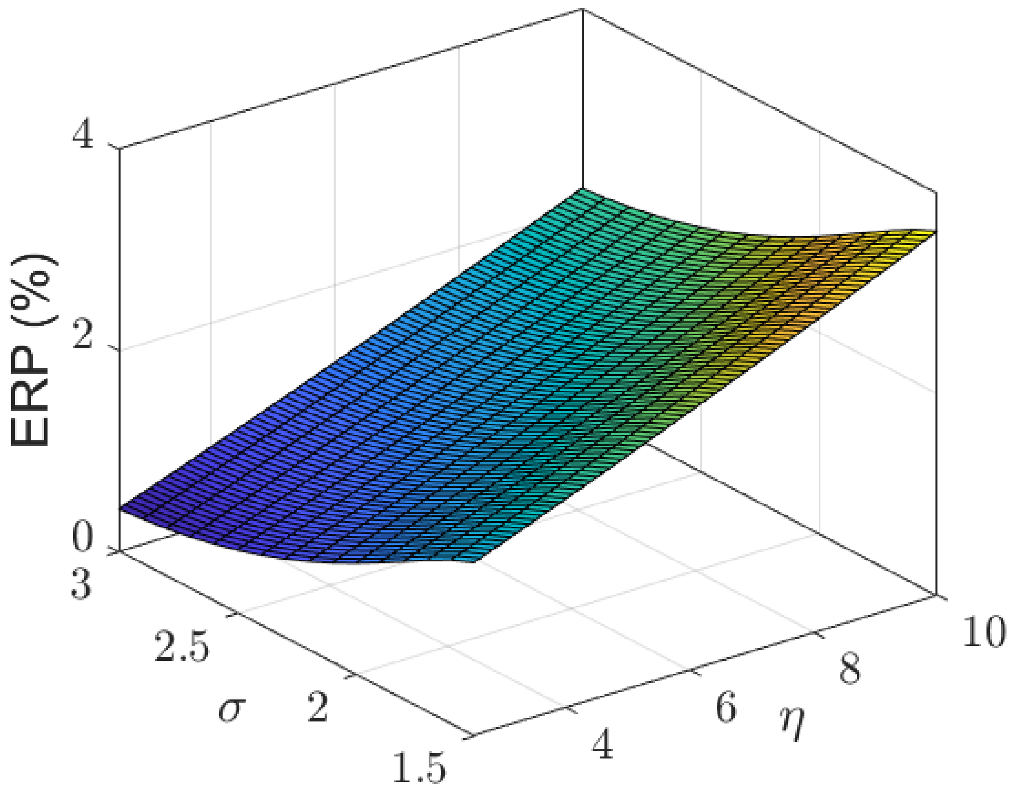
Notes to Table: This table displays the parameterization used for the numerical computations of the model equilibrium presented in Section 3. ρ_w, ρ_π are the estimated autocorrelation coefficients of the first order autoregressive processes of annual log per capita U.S. income ($w_t = \log W_t$) and log price index ($\pi_t = \log\,(W_t/C_t)$, $C_t =$ annual per capita U.S. consumption), that is, $w_t = \rho_w w_{t-1} + \varepsilon_{wt}$ and $\pi_t = \rho_\pi \pi_{t-1} + \varepsilon_{\pi t}$. Φ_w, Φ_π and $\Phi_{w\pi}$ are the volatilities and covariance of the estimated income and price index shocks ε_{wt} and $\varepsilon_{\pi t}$. The intertemporal elasticity of substitution (η^{-1}) and intratemporal product elasticity of substitution are calibrated from the structural estimation of the model (in Table 3). The production and sectoral parameters are calibrated for the U.S. manufacturing sector for 1958-2016 using the NBER-CES (annual) data. The output elasticities of capital and material inputs ψ_K, ψ_H are based on estimation of the Cobb-Douglas production function in (2.8), while the autocorrelation coefficients of log of productivity shocks and logs of materials price index and investment price index (ρ_a, ρ_x, ρ_z), as well as the covariances of industry productivity and input price shocks with w_t and π_t ($\Phi_{aw}, \Phi_{a\pi}, \Phi_{xw}, \Phi_{x\pi}, \Phi_{zw}, \Phi_{z\pi}$) are estimated from the first order autoregressive processes of log productivity and material inputs during 1958-2016.

Figure 1: Product elasticity of substitution (σ), risk aversion (γ), and equity risk premium



Notes to Figure: This figure graphically displays, through three-dimensional plots, the relation of equilibrium equity risk premium for the consumer durables manufacturing industry, with various combinations of consumer preference parameters: risk aversion (γ), (inverse of) intertemporal elasticity of substitution (η) and intratemporal elasticity of substitution (σ). The sample period is 1958-2016 (annual) and data sources are described in text.

Figure 2: Product elasticity of substitution (σ), intertemporal elasticity of substitution (η^{-1}), and equity risk premium



Notes to Figure: This figure graphically displays, through three-dimensional plots, the relation of equilibrium equity risk premium for the consumer durables manufacturing industry, with various combinations of consumer preference parameters: risk aversion (γ), (inverse of) intertemporal elasticity of substitution (η) and intratemporal elasticity of substitution (σ). The sample period is 1958-2016 (annual) and data sources are described in text.

Online Appendix

A.1 Derivation of Optimal Consumption and Portfolio Policies

The representative consumer-investor's (CI's) optimization problem at any t is to

$$\max_{\mathbf{c}_t,\mathbf{q}_{t+1}} \mathcal{U}_t, \text{ s.t.,} \tag{A1.1}$$

$$\mathbf{p}_t \cdot \mathbf{c}_t \leq \mathbf{q}_t \cdot (\mathbf{d}_t + \mathbf{s}_t) - \mathbf{q}_{t+1} \cdot \mathbf{s}_t \equiv W_t. \tag{A1.2}$$

The Lagrangian for with respect to (A1.1)-(A1.2) is

$$\max_{\mathbf{c}_t,\mathbf{q}_{t+1}} \mathcal{U}_t + \chi_t \left[W_t - \mathbf{p}_t \cdot \mathbf{c}_t\right], \ W_t = \mathbf{q}_t \cdot (\mathbf{d}_t + \mathbf{s}_t) - \mathbf{q}_{t+1} \cdot \mathbf{s}_t, \tag{A1.3}$$

where Lagrange multiplier for the budget constraint $\chi_t > 0$ since preferences are strictly increasing in consumption and the budget constraint (A1.2) will be binding in optimum. Using concavity of the objective and convexity of the constraint, the optimal consumption and portfolio policies can be characterized through a two-step process, where the optimal consumption vector $\mathbf{c}_t$ is first determined as a function of available consumption expenditure W_t, and the portfolio $\mathbf{q}_{t+1}$ is then determined taking as given the optimal consumption policy.

Then, using the definition of the consumption basket C_t in (2.2), the first order optimality conditions for $c_{jt}, j = 1, ...J$, can be written

$$[(1-\alpha)(1-\eta)](C_t)^{\frac{1-\eta\sigma}{\sigma}}(c_{jt})^{-\frac{1}{\sigma}}\phi_j = \chi_t p_{jt}. \tag{A1.4}$$

Isolating c_{jt} in (A1.4) and multiplying both sides by p_{jt} yields

$$p_{jt}c_{jt} = \chi_t^{-\sigma}(p_{jt})^{1-\sigma}(C_t)^{-(1-\eta\sigma)}(\phi_j)^{\sigma}[(1-\alpha)(1-\eta)]^{\sigma}. \tag{A1.5}$$

Then recognizing that $W_t = \sum_j p_{jt}c_{jt}$ and $P_t = \left[\sum_{j=1}^{J}(\phi_j)^{\sigma}(p_{jt})^{1-\sigma}\right]^{1/(1-\sigma)}$, summing both sides of (A1.5) over j allows one to solve for the Lagrange multiplier as

$$\chi_t = \left(\frac{W_t}{P_t}\right)^{-\frac{1}{\sigma}} P_t^{-1}(C_t)^{\frac{1-\eta\sigma}{\sigma}}[(1-\alpha)(1-\eta)]. \tag{A1.6}$$

Substituting for χ_t in (A1.4) and rearranging terms then gives the optimal consumption functions in (2.4), that is,

$$c_{jt}(\mathbf{p}_t^c, W_t) = \frac{W_t}{P_t}\left[\frac{P_t\phi_j}{p_{jt}}\right]^{\sigma}, \ j = 1, ..J. \tag{A1.7}$$

Now (A1.7) implies

$$(C_t)^{\frac{\sigma-1}{\sigma}} = \sum_j \phi_j (c_{jt})^{\frac{\sigma-1}{\sigma}} = \left(\frac{W_t}{P_t}\right)^{\frac{\sigma-1}{\sigma}} (P_t)^{\sigma-1} \left(\sum_j (\phi_j)^{\sigma} (p_{jt})^{1-\sigma}\right). \qquad \text{(A1.8)}$$

But since $\sum_j (\phi_j)^{\sigma}(p_{jt})^{1-\sigma} = (P_t)^{1-\sigma}$, (A1.8) yields $C_t = \frac{W_t}{P_t}$.

Next, conditional on optimal $\mathbf{c}_t$ and, hence, $C_t = \frac{W_t}{P_t}$, the derivation of the optimal portfolio condition (2.6) with Epstein and Zin (1989) preferences is standard using straightforward application of arguments in Epstein and Zin (1989). ∎

A.2 Derivation of Optimal Intermediate Goods Policies

We derive the optimal demand for material intermediate goods. The derivation for optimal investment intermediate goods is analogous.

The firm's objective is to maximize effective materials input

$$H_t^j = \left[\sum_{n=J_c+1}^{J_h} \varphi_{nj}^h (H_{nt}^j)^{(\zeta_j^h-1)/\zeta_j^h}\right]^{\zeta_j^h/(\zeta_j^h-1)} \qquad \text{(A2.1)}$$

subject to a fixed materials cost Υ_{ht}^j. Hence, the Lagrangian is

$$\max_{H_{nt}^j} \left[\sum_{n=J_c+1}^{J_h} \varphi_{nj}^h (H_{nt}^j)^{(\zeta_j^h-1)/\zeta_j^h}\right]^{\zeta_j^h/(\zeta_j^h-1)} + \chi_{ht}^j \left[\Upsilon_{ht}^j - \sum_{n=J_c+1}^{J_h} p_t^n H_{nt}^j\right]. \qquad \text{(A2.2)}$$

This yields the optimality conditions for $H_{nt}^j, n = 1, ...J$

$$(H_t^j)^{\frac{1}{(\zeta_j^h-1)}} (H_{nt}^j)^{-\frac{1}{\zeta_j^h}} \varphi_{nj} = \chi_{ht}^j p_t^n, \qquad \text{(A2.3)}$$

which implies that

$$p_t^n H_{nt}^j = (\chi_{ht}^j)^{-\zeta_j^h} (H_t^j)^{\frac{\zeta_j^h}{(\zeta_j^h-1)}} (p_t^n)^{1-\zeta_j^h} (\varphi_{nj})^{\zeta_j^h}. \qquad \text{(A2.4)}$$

Summing both sides of (A2.4) over n gives

$$\chi_{ht}^j = (\Upsilon_{ht}^j)^{-\frac{1}{\zeta_j^h}} (H_t^j)^{\frac{1}{(\zeta_j^h-1)}} \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{1/\zeta_j^h}. \qquad \text{(A2.5)}$$

Substituting (A2.5) in (A2.3) yields,

$$H_{nt}^j = (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{-\zeta_j^h} \Upsilon_{ht}^j \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{-1}. \qquad \text{(A2.6)}$$

Now put $X_t^j = \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{1/(1-\zeta_j^h)}$ so that $X_t^j = \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{-1} = (X_t^j)^{\zeta_j^h - 1}$. Hence,

$$\Upsilon_{ht}^j \left[\sum_{n=J_c+1}^{J_h} (\varphi_{nj}^h)^{\zeta_j^h} (p_t^n)^{1-\zeta_j^h}\right]^{-1} = \left(\frac{\Upsilon_{ht}^j}{X_t^j}\right) (X_t^j)^{\zeta_j^h}. \tag{A2.7}$$

Let us now conjecture that $\left(\frac{\Upsilon_{ht}^j}{X_t^j}\right) = H_t^j$, so that (A2.6)-(A2.7) together yield,

$$H_{nt}^j = (\varphi_{nj}^h)^{\zeta_j^h} \left[\frac{p_t^n}{X_t^j}\right]^{-\zeta_j^h} H_t^j. \tag{A2.8}$$

Then using (A2.8) in $\Upsilon_{ht}^j = \sum_{n=J_c+1}^{J_h} p_t^n H_{nt}^j$ indeed verifies that $\frac{\Upsilon_{ht}^j}{X_t^j} = H_t^j$. ∎

A.3 Proof of Proposition 1: As in the text, we suppress the notation for sectors (unless necessary) for expositional ease. Using the Bellman-representation (2.16), along any competitive equilibrium path at any t, conditional on $\Omega_t = (W_t, P_t, A_t, X_t, Z_t, K_t)$, the optimization problem for the typical competitive firm is

$$V_t(\Omega_t) = \max_{I_t, H_t \geq 0} D_t + \mathbb{E}_t \left[V_{t+1}(\Omega_{t+1})\right], \text{ s.t.,} \tag{A3.1}$$

$$D_t = p_t Y_t - X_t H_t - O(I_t, K_t), \tag{A3.2}$$

$$p_t = \phi \left(W_t P_t^{\sigma-1}\right)^{1/\sigma} (Y_t)^{-1/\sigma}, \tag{A3.3}$$

$$Y_t = F(K_t, H_t, A_t) = A_t (K_t)^{\psi_K} (H_t)^{\psi_H}, \tag{A3.4}$$

$$O(I_t, K_t) = Z_t I_t + 0.5 \upsilon \left(\frac{I_t}{K_t}\right)^2 K_t, \tag{A3.5}$$

$$K_{t+1} = (1-\delta) K_t + I_t. \tag{A3.6}$$

Taking the equilibrium sectoral price p_t as given, optimization with respect to H_t then yields

$$p_t F_H(K_t, H_t, A_t) = X_t. \tag{A3.7}$$

Next, the optimal (interior) I_t satisfies

$$0 = \frac{\partial D_t}{\partial I_t} + \mathbb{E}_t \left[\frac{\partial V_{t+1}(\Omega_{t+1})}{\partial K_{t+1}}\right]. \tag{A3.8}$$

But from (A3.2) and (A3.5),

$$\frac{\partial D_t}{\partial I_t} = -O_I(I_t, K_t) = -\left[Z_t + \upsilon \left(\frac{I_t}{K_t}\right)\right]. \tag{A3.9}$$

And using the intertemporal envelope theorem (that sets the indirect effects of ∂K_{t+1} on the optimally chosen I_{t+1} and H_{t+1} to zero), along the competitive equilibrium path

$$\begin{aligned}\frac{\partial V_{t+1}(\Omega_{t+1})}{\partial K_{t+1}} &= p_{t+1}F_K(K_{t+1},H_{t+1},A_{t+1}) - O_K(I_{t+1},K_{t+1}) - (1-\delta)\frac{\partial D_{t+1}}{\partial I_{t+1}} \\ O_K(I_{t+1},K_{t+1}) &= -0.5\upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)^2, \frac{\partial D_{t+1}}{\partial I_{t+1}} = -O_I(I_{t+1},K_{t+1}). \qquad \text{(A3.10)}\end{aligned}$$

(A3.8) then yields,

$$\begin{aligned}\left[Z_t + \upsilon\left(\frac{I_t}{K_t}\right)\right] &= \mathbb{E}_t\left[\Omega_{t,t+1}\left\{p_{t+1}F_K(K_{t+1},H_{t+1},A_{t+1}) + 0.5\upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + \right.\right. \\ &\left.\left. (1-\delta)\left[Z_{t+1} + \upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)\right]\right\}\right], \qquad \text{(A3.11)}\end{aligned}$$

where $K_{t+1} = K_t(1-\delta) + I_t$. The equilibrium product price is then obtained from (A3.3) as

$$p_t = \phi\left(\frac{W_t}{P_t}\right)^{1/\sigma} P_t\left[F(K_t,H_t,A_t)\right]^{-1/\sigma}. \qquad \text{(A3.12)}$$

Finally, the equilibrium asset market restriction in Equation (2.21) follows from Equation (2.7). ∎

As in the text, we suppress the notation for sectors (unless necessary) for expositional ease. ∎

B. Equilibrium Computations

B.1 Steady State Computations

We will denote the deterministic steady state with bars. Note that in the deterministic steady state the pricing kernel is $\bar{\Lambda} = \alpha$ because $W_{t+1} = W_t = \bar{W}, P_{t+1} = P_t = \bar{P}$, and $\mathcal{U}_t = \mathcal{U}_{t+1}$. Now, in the deterministic steady state, the equilibrium input choice and and capital stock $(\bar{H},\bar{K})$ are fixed (and hence $\bar{I} = \delta\bar{K}$). These quantities are derived from the optimality conditions (2.18) and (2.19) in the text. In the steady state, the Euler conditions are

$$\begin{aligned}\bar{p}F(\bar{K},\bar{H},\bar{A}) &= \bar{X}, \qquad \text{(B.1.1)} \\ -(\bar{Z}+\upsilon\delta) + \alpha\left[\bar{p}F(\bar{K},\bar{H},\bar{A}) + 0.5\upsilon(\delta)^2 + (1-\delta)(\bar{Z}+\upsilon\delta)\right] &= 0, \qquad \text{(B.1.2)}\end{aligned}$$

where the steady state sector prices and sales are

$$\begin{aligned}\bar{p} &= \phi\bar{W}^{1/\sigma}\bar{P}^{\frac{\sigma-1}{\sigma}}[F(\bar{K},\bar{H},\bar{A})]^{-1/\sigma}, \qquad \text{(B.1.3)} \\ \bar{\Psi} &= \phi\bar{W}^{1/\sigma}\bar{P}^{\frac{\sigma-1}{\sigma}}[F(\bar{K},\bar{H},\bar{A})]^{(\sigma-1)/\sigma} \qquad \text{(B.1.4)}\end{aligned}$$

Noting that $F(\bar{K},\bar{H},\bar{A}) = \bar{A}(\bar{K})^{\psi}(\bar{H})^{\psi}$, $F(\bar{K},\bar{H},\bar{A}) = \psi F(\bar{K},\bar{H},\bar{A})/H$, $F(\bar{K},\bar{H},\bar{A}) = \psi F(\bar{K},\bar{H},\bar{A})/K$, and substituting (B.1.3)-(B.1.4) in (B.1.1)-(B.1.2) yields the system of equations

$$\bar{H} = \left[(\phi\psi)^{\sigma}\bar{W}\bar{P}^{\sigma-1}(\bar{A})^{\sigma-1}(\bar{K})^{(\sigma-1)\psi}(\bar{X})^{-\sigma}\right]^{1/\nu}, \tag{B.15}$$

$$\bar{K} = \left[\left(\frac{\alpha\phi\psi}{e}\right)^{\sigma}\bar{W}\bar{P}^{\sigma-1}(\bar{A})^{\sigma-1}(\bar{H})^{(\sigma-1)\psi}\right]^{1/\nu}, \tag{B.1.6}$$

where $\nu \equiv \psi + \sigma(1-\psi)$, $\nu \equiv \psi + \sigma(1-\psi)$, and $e \equiv \bar{Z} + \left(\frac{\upsilon\delta}{2}\right)(2 - \alpha(2+\delta))$.

To solve for $(\bar{H}, \bar{K})$ analytically, it is convenient to take the log of both sides of (B.15)-(B.1.6). Using small letters to express logs, setting $\bar{\pi} = \log(\bar{P})$), and solving for $\bar{H}$we get

$$\begin{aligned}\bar{H} &= \exp\left((\varsigma)^{-1}\left[\sigma(\log\phi + \log\psi - \bar{x}) + (\sigma-1)\left(\frac{\psi}{\nu}\right)\times\right.\right.\\ &\quad \left.\left.(\log\alpha + \log\phi + \log\psi - \log e) + \left(1 + (\sigma-1)\left(\frac{\psi}{\nu}\right)\right)[\bar{w} + (\sigma-1)(\bar{a}+\bar{\pi})]\right]\right)\\ &= \left[(\phi\psi)^{\sigma}(\bar{W}\bar{P}^{\sigma-1}(\bar{A})^{\sigma-1})^{n}\left(\frac{\alpha\phi\psi}{e}\right)^{\varsigma}(\bar{X})^{-\sigma}\right]^{1/o},\end{aligned} \tag{B.1.7}$$

where $\varsigma \equiv (\nu x - (\sigma-1)^2\psi(\psi/\nu))$, $\varsigma \equiv (\sigma-1)(\psi/\nu)$, and $n \equiv 1 + \varsigma$. $\bar{K}$is then recovered from substituting (B.1.7) in (B.1.6).

B.2 Analytic Approximations

B.2.1 Pricing Kernel

From (2.5), the log of the real pricing kernel $\lambda_{t+1} \equiv \log(\Lambda_{t,t+1})$ is

$$\lambda_{t+1} = \theta\log\alpha - \eta\theta\left[g_{w,t+1} - g_{\pi,t+1}\right] + (\theta-1)r_{c,t+1}, \tag{B.2.1}$$

where $g_{w,t+1}$ and $g_{\pi,t+1}$ are the log growth rates of income and aggregate price, respectively, and $r_{c,t+1} \equiv \log(R_{C,t+1})$. Using the Campbell and Shiller (1988) log-linearization approach, we can represent $r_{c,t+1}$ as

$$r_{c,t+1} = f_0 + f_1 z_{c,t+1} - z_{ct} + g_{w,t+1} - g_{\pi,t+1}, \tag{B.2.2}$$

where z_{ct} is the log price-consumption ratio. Here f_0 and f_1 are approximating constants that depend on the unconditional mean of z_{ct}, say, z_c. (In our numerical computations, we take z_c to be the mean of simulated $(w_t - p_t)$.) Indeed,

$$f_0 = \log(1 + \exp(z_c)) - f_1 z_c;\; f_1 = \frac{\exp(z_c)}{1 + \exp(z_c)}. \tag{B.2.3}$$

Exploiting the (log form of) the Euler condition (2.5), in the setting at hand z_{ct} is a linear function of the logs of the aggregate state variables W_t and P_t, namely,

$$z_{ct} = \kappa_{c0} + \kappa_{cw} w_t + \kappa_{c\pi} \pi_t. \tag{B.2.4}$$

Since $g_{w,t+1} = (\rho_w - 1)w_t + \varepsilon^w_{t+1}$ and $g_{\pi,t+1} = (\rho_\pi - 1)\pi_t + \varepsilon^\pi_{t+1}$, substitution of (B.2.2) and (B.2.4) in (B.2.1) allows one to write

$$\lambda_{t+1} = B_0 + B_w w_t + B_\pi \pi_t + b_w \varepsilon_{w,t+1} + b_\pi \varepsilon_{\pi,t+1}, \tag{B.2.5}$$

where

$$\begin{aligned} B_0 &= \theta \log \alpha; \\ B_w &\equiv (\rho_w - 1)\left[1 - \eta\theta + (\theta - 1) f_1 \kappa_{cw}\right] - (\theta - 1)\kappa_{cw}; \\ B_\pi &\equiv (\rho_\pi - 1)\left[\eta\theta - 1 - (\theta - 1) f_1 \kappa_{c\pi}\right] + (\theta - 1)\kappa_{c\pi}; \\ b_w &\equiv -\eta\theta + (\theta - 1)[f_1 \kappa_{cw} + 1]; \\ b_\pi &\equiv \eta\theta - (\theta - 1)[f_1 \kappa_{c\pi} + 1]. \end{aligned} \tag{B.2.6}$$

We can then obtain the coefficients of z_{ct} in (B.2.4) through the method of undetermined coefficients. The Euler condition (2.5) is

$$\begin{aligned} 1 &= \mathbb{E}_t[\exp(\lambda_{t+1} + \pi_t - \pi_{t+1} + r_{c,t+1})] \\ &= \mathbb{E}_t[\exp(\theta \log \alpha - \eta\theta\left[g_{w,t+1} - g_{\pi,t+1}\right] + \theta r_{c,t+1} + \pi_t - \pi_{t+1})] \end{aligned} \tag{B.2.7}$$

Since (B.2.7) must hold for all values of the state variables, all terms involving w_t and π_t must satisfy

$$w_t \theta[(\rho_w - 1)(1 - \eta) + \kappa_{cw}(f_1(\rho_w - 1) - 1)] = 0, \tag{B.2.8}$$

$$\pi_t \left\{\theta[(\rho_\pi - 1)(\eta - 1) + \kappa_{c\pi}(f_1(\rho_\pi - 1) - 1)] + 1 - \rho_\pi\right\} = 0. \tag{B.2.9}$$

From (B.2.8)-(B.2.9), it follows

$$\kappa_{cw} = \frac{(\rho_w - 1)(\eta - 1)}{f_1(\rho_w - 1) - 1}, \kappa_{c\pi} = \frac{(\rho_\pi - 1)[\theta(1 - \eta) + 1]}{\theta(f_1(\rho_\pi - 1) - 1)}. \tag{B.2.10}$$

And to ensure that the constant terms in (B.2.7) equal zero, from (B.2.2) and (B.2.4), the coefficient κ_{c0} is calculated as, $\kappa_{c0} = \frac{\log \alpha + f_0}{1 - f_1}$. Finally, note that the log of the nominal pricing kernel $m_{t+1} = \lambda_{t+1} + \pi_t - \pi_{t+1}$ is

$$m_{t+1} = B_0 + B_w w_t + (B_\pi + (1 - \rho_\pi))\pi_t + b_w \varepsilon_{w,t+1} + b_\pi \varepsilon_{\pi,t+1}. \tag{B.2.11}$$

B.2.2 Approximations of Industry Equilibrium Conditions

Note that the optimality condition for materials inputs (2.18) can be written

$$
\begin{aligned}
0 &= \frac{\psi_H \Psi_t}{H_t} - X_t, \text{ where} \\
\Psi_t &= \phi W_t^{1/\sigma} P_t^{\frac{\sigma-1}{\sigma}} [A_t (K_t)^{\psi_K} (H_t)^{\psi_H}]^{\frac{(\sigma-1)}{\sigma}} \qquad \text{(B.2.12)}
\end{aligned}
$$

Hence, log-linearization of the optimality condition around the steady state implies

$$
\left(\frac{\psi_H \bar{\Psi}}{\bar{H}}\right) \left[\sigma + \hat{W}_t + (\sigma - 1)\left\{\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t\right\} - \nu_H \hat{H}_t\right] - \sigma \bar{X}(1 + \hat{X}_t) = 0, \qquad \text{(B.2.13)}
$$

where $\nu_H \equiv \psi_H + \sigma(1 - \psi_H)$ has been defined above. But using the fact that in the steady state $\psi_H \bar{\Psi}(\bar{H})^{-1} = \bar{X}$, $(B.2.13)$ gives

$$
\left(\frac{\psi_H \bar{\Psi}}{\bar{H}}\right) \left[\hat{W}_t + (\sigma - 1)\left\{\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t\right\} - \nu_H \hat{H}_t\right] - \sigma \bar{X} \hat{X}_t = 0 \qquad \text{(B.2.14)}
$$

Dividing through by $\psi_H \bar{\Psi}(\bar{H})^{-1}$ and noting that $\bar{X}/(\psi_H \bar{\Psi}(\bar{H})^{-1}) = 1$ and rearranging terms then yields

$$
\hat{H}_t = (\nu_H)^{-1} \left[\hat{W}_t + (\sigma - 1)\left\{\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t\right\} - \sigma \hat{X}_t\right]. \qquad \text{(B.2.15)}
$$

To derive the log-linearized form of equilibrium investment, we use the fact that

$$
I_t = K_{t+1} - (1 - \delta) K_t
$$

and reformulate the optimization problem for investment (in the standard way) as the choice of K_{t+1} at t. Noting that

$$
\frac{\partial \Psi_{t+1}}{\partial K_{t+1}} = p_{t+1} F_K(K_{t+1}, H_{t+1}, A_{t+1}) = \psi_K \frac{\Psi_{t+1}}{K_{t+1}}, \qquad \text{(B.2.16)}
$$

the investment Euler condition (2.19) can be written

$$
Z_t + \upsilon\left(\frac{I_t}{K_t}\right) = \mathbb{E}_t\left[M_{t,t+1}\left\{\psi_K \frac{\Psi_{t+1}}{K_{t+1}} + \left(\frac{\upsilon}{2}\right)\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 + (1 - \delta)\left(Z_{t+1} + \upsilon\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right\}\right]. \qquad \text{(B.2.17)}
$$

To derive the equilibrium law of motion for capital stock, one looks for a second order difference equation in $\hat{K}_t$. We will log-linearize (B.2.17) and then use the log-linearization of the investment transition equation to get:

$$
\hat{I}_t = (\delta)^{-1}\left(\hat{K}_{t+1} - \hat{K}_t(1 - \delta)\right). \qquad \text{(B.2.18)}
$$

In (B.2.18), we use the fact that $\bar{I} = \delta \bar{K}$, so that $\bar{K}/\bar{I} = 1/\delta$. Then log-linearization of

(B.2.17) implies (noting that $\bar{M} = \alpha$) :

$$\begin{aligned}\bar{Z}(1+\hat{Z}_t) + \upsilon\delta(1+\hat{I}_t - \hat{K}_t) \;\; = \;\; & \alpha \mathbb{E}_t \Bigg[\psi_K \frac{\bar{\Psi}}{\bar{K}} (1 + \hat{M}_{t,t+1} + \hat{\Psi}_{t+1} - \hat{K}_{t+1}) + (1-\delta)\bar{Z} \times \\ & (1 + \hat{Z}_{t+1} + \hat{M}_{t,t+1}) \; + \left(\frac{\upsilon}{2}\right) (\delta)^2 (1 + \hat{M}_{t,t+1} + 2(\hat{I}_{t+1} - \hat{K}_{t+1})) \; + \\ & \upsilon\delta(1-\delta)(1 + \hat{M}_{t,t+1} + (\hat{I}_{t+1} - \hat{K}_{t+1})) \Bigg] \qquad \text{(B.2.19)}\end{aligned}$$

Rearranging terms, the RHS of (B.2.19) can be written

$$\begin{aligned}& \alpha \mathbb{E}_t \Bigg[(1 + \hat{M}_{t,t+1}) \left(\psi_K \frac{\bar{\Psi}}{\bar{K}} + (1-\delta)\bar{Z} + \left(\frac{\upsilon\delta}{2}\right)(2-\delta) \right) + \\ & (1-\delta)\bar{Z}\hat{Z}_{t+1} + \psi_K \frac{\bar{\Psi}}{\bar{K}} (\hat{\Psi}_{t+1} - \hat{K}_{t+1}) + \upsilon\delta(\hat{I}_{t+1} - \hat{K}_{t+1}) \Bigg]. \qquad \text{(B.2.20)}\end{aligned}$$

But in the steady state, the Euler for investment is

$$Z + \upsilon\delta = \alpha \left[\psi_K \frac{\bar{\Psi}}{\bar{K}} + Z(1-\delta) + \left(\frac{\upsilon\delta}{2}\right)(2-\delta) \right]. \qquad \text{(B.2.21)}$$

Combining (B.2.20) and (B.2.21), we can write the log-linearized Euler (B.2.17) as

$$\begin{aligned}0 \;\; = \;\; & \mathbb{E}_t \Bigg[\hat{M}_{t,t+1} u^{M,} + \psi_K \frac{\bar{\Psi}}{\bar{K}} (\hat{\Psi}_{t+1} - \hat{K}_{t+1}) + \upsilon\delta \left\{ (\hat{I}_{t+1} - \hat{K}_{t+1}) - \alpha^{-1}(\hat{I}_t - \hat{K}_t) \right\} + \\ & \bar{Z} \left\{ (1-\delta)\hat{Z}_{t+1} - \alpha^{-1}\hat{Z}_t \right\} \Bigg], \qquad \text{(B.2.22)}\end{aligned}$$

where $u^{M,} \equiv \left(\psi_K \frac{\bar{\Psi}}{\bar{K}} + (1-\delta)\bar{Z} + \left(\frac{\upsilon\delta}{2}\right)(2-\delta) \right)$. Now note from (B.2.12) that

$$\Psi_{t+1} = \phi W_{t+1}^{1/\sigma} P_{t+1}^{\frac{\sigma-1}{\sigma}} [A_{t+1}(K_{t+1})^{\psi_K} (H_{t+1})^{\psi_H}]^{(\sigma-1)/\sigma}$$

Hence, log-linearization gives

$$\begin{aligned}\hat{\Psi}_{t+1} \;\; &= \;\; \left(\frac{\phi}{\sigma}\right) \left[\hat{W}_{t+1} + (\sigma-1)\{\hat{P}_{t+1} + \hat{A}_{t+1} + \psi_K \hat{K}_{t+1} + \psi_H \hat{H}_{t+1}\} \right], \qquad \text{(B.2.23)} \\ \hat{H}_{t+1} \;\; &= \;\; (\nu_H)^{-1} \left[\hat{W}_{t+1} + (\sigma-1) \left\{ \hat{P}_{t+1} + \hat{A}_{t+1} + \psi_K \hat{K}_{t+1} \right\} - \sigma \hat{X}_{t+1} \right], \qquad \text{(B.2.24)}\end{aligned}$$

where (B.2.24) follows from (B.2.15). However, from (B.2.18)

$$\hat{I}_{t+i} - \hat{K}_{t+i} = (\delta)^{-1} \left(\hat{K}_{t+i} - \hat{K}_t \right), i = 0, 1$$

Substituting in (B.2.22) then yields

$$0 = \mathbb{E}_t \Big[\hat{M}_{t,t+1} u^{M,} + \left(\frac{\phi \psi_K}{\sigma} \right) \frac{\bar{\Psi}}{\bar{K}} \{ (\hat{W}_{t+1} + (\sigma - 1)(\hat{P}_{t+1} + \hat{A}_{t+1}))(1 + u^{H,}) - u^{H,} \sigma \hat{X}_{t+1} \} + u_2^{K,} \hat{K}_{t+2} + u_1^{K,} \hat{K}_{t+1} + u_0^{K,} \hat{K}_t + \bar{Z} \left\{ (1-\delta) \hat{Z}_{t+1} - \alpha^{-1} \hat{Z}_t \right\} \Big] \quad \text{(B.2.25)}$$

where $u^{H,} = (\nu_H)^{-1} \psi_H, u_2^{K,} = \upsilon$, $u_0^{K,} = \left(\frac{\upsilon}{\alpha} \right)$, and

$$u_1^{K,} = \left(\frac{\psi_K \bar{\Psi}}{\bar{K}} \left(\frac{\phi}{\sigma} (\sigma - 1) \psi_K (1 + u^{H,}) - \sigma \right) - \frac{\upsilon(1+\alpha)}{\alpha} \right). \quad \text{(B.2.26)}$$

We now use the method of undetermined coefficients to ensure that the right hand side of (B.2.25) is zero for all values of the state variables. We will write $k_t = \log(K_t)$. Recall that

$$\hat{M}_{t,t+1} = \lambda_{t+1} + \pi_t - \pi_{t+1} - \log \alpha.$$

Moreover, $\hat{W}_{t+1} = w_{t+1} - \bar{w}$, where $\bar{w} = \log \bar{W}$. Hence, we have

$$\begin{aligned} \hat{W}_{t+1} &= \rho_w w_t + \varepsilon_{t+1}^w - \bar{w}; \\ \hat{P}_{t+1} &= \rho_\pi \pi_t + \varepsilon_{t+1}^\pi - \bar{\pi} \\ \hat{A}_{t+1} &= \rho_a a_t + \varepsilon_{a,t+1} - \bar{a} \\ \hat{X}_{t+1} &= \rho_x x_t + \varepsilon_{x,t+1} - \bar{x}, \\ \hat{Z}_{t+1} &= \rho_z z_t + \varepsilon_{z,t+1} - \bar{z} \end{aligned}$$

and finally $\hat{K}_{t+i} = k_{t+i} - \bar{k}, i = 0, 1, 2$. Then from (B.2.11)

$$\mathbb{E}_t \left[\hat{M}_{t,t+1} u^{M,} \right] = u^{M,} [B_w w_t + (B_\pi \pi_t + (1 - \rho_\pi) \pi_t + (\theta - 1) \log \alpha] \quad \text{(B.2.27)}$$

Similarly, the expectation of the second term in (B.2.25) is

$$\left(\frac{\phi \psi_K}{\sigma} \right) \frac{\bar{\Psi}}{\bar{K}} \left\{ \left([\rho_w w_t + (\sigma - 1)(\rho_\pi \pi_t + \rho_a a_t)](1 + u^{H,}) - u^{H,} \sigma \rho_x x_t \right) - \left[(\bar{w} + (\sigma - 1)(\bar{\pi} + \bar{a}))(1 + u^{H,}) - u^{H,} \sigma \bar{x} \right] \right\}. \quad \text{(B.2.28)}$$

Next, we conjecture (and subsequently verify) that k_{t+1} is an affine function of the form

$$k_{t+1} = \xi_0 + \xi_w w_t + \xi_\pi \pi_t + \xi_a a_t + \xi_x x_t + \xi_z z_t + \xi_k k_t, \quad \text{(B.2.29)}$$

Hence

$$\mathbb{E}_t \left[u_2^{K,} \hat{K}_{t+2} + u_1^{K,} \hat{K}_{t+1} + u_0^{K,} \hat{K}_t \right] =$$

$$k_t \left[u_2^{K,}(\xi_k)^2 + u_1^{K,}\xi_k + u_0^{K,} \right] + w_t \left[\xi_w \{ u_2^{K,}(\rho_w + \xi_k) + u_1^{K,} \} \right] +$$
$$\pi_t \left[\xi_\pi \{ u_2^{K,}(\rho_\pi + \xi_k) + u_1^{K,} \} \right] + a_t \left[\xi_a \{ u_2^{K,}(\rho_a + \xi_k) + u_1^{K,} \} \right] + x_t \left[\xi_x \{ u_2^{K,}(\rho_x + \xi_k) + \right.$$
$$\left. u_1^{K,} \} \right] + z_t \left[\xi_z \{ u_2^{K,}(\rho_z + \xi_k) + u_1^{K,} \} \right] - \xi_0 \left[u_2^{K,}(1 + \xi_k) + u_1^{K,} \right] -$$
$$[\xi_w \bar{w} + \xi_\pi \bar{\pi} + \xi_a \bar{a} + \xi_x \bar{x} + \xi_k \bar{k}].$$

To ensure that terms multiplying k_t equal zero, the following quadratic equation in ξ_k must be satisfied

$$u_2^{K,}(\xi_k)^2 + u_1^{K,}\xi_k + u_0^{K,} = 0$$

so that

$$\xi_k = -\frac{u_1^{K,}}{2u_2^{K,}} \pm \frac{\sqrt{(u_1^{K,})^2 - 4u_2^{K,}u_0^{K,}}}{2u_2^{K,}} \tag{B.2.31}$$

In the standard way, the root with $\mid \xi_k \mid < 1$ will be chosen. Next collecting terms for w_t and requiring them to be zero, we have

$$w_t \left[\xi_w \{ u_2^{K,}(\rho_w + \xi_k) + u_1^{K,} \} + u^{M,}B_w + \left(\frac{\phi \psi_K \bar{\Psi}}{\sigma \bar{K}} \right) \rho_w (1 + u^{H,}) \right] = 0 \tag{B.2.32}$$

so that

$$\xi_w = -\frac{\left(\frac{\phi \psi_K \bar{\Psi}}{\sigma \bar{K}} \right) \rho_w (1 + u^{H,}) + u^{M,}B_w}{u_2^{K,}(\rho_w + \xi_k) + u_1^{K,}}, \tag{B.2.33}$$

which is well defined since ξ_k is known from (B.2.31). Similar calculations then show

$$\xi_\pi = -\frac{\left(\frac{\phi \psi_K \bar{\Psi}}{\sigma \bar{K}} \right) (\sigma - 1)\rho_\pi (1 + u^{H,}) + u^{M,}(B_\pi + (1 - \rho_\pi))}{u_2^{K,}(\rho_\pi + \xi_k) + u_1^{K,}}, \tag{B.2.34}$$

$$\xi_a = -\frac{\left(\frac{\phi \psi_K \bar{\Psi}}{\sigma \bar{K}} \right) (\sigma - 1)\rho_a (1 + u^{H,})}{u_2^{K,}(\rho_a + \xi_k) + u_1^{K,}}, \tag{B.2.35}$$

$$\xi_x = \frac{\left(\frac{\phi \psi_K \bar{\Psi}}{\sigma \bar{K}} \right) u^{H,}\sigma \rho_x}{u_2^{K,}(\rho_x + \xi_k) + u_1^{K,}}, \tag{B.2.36}$$

$$\xi_z = \frac{1 - \alpha \bar{Z}(1 - \delta)\rho_z}{\alpha \left[u_2^{K,}(\rho_z + \xi_k) + u_1^{K,} \right]}. \tag{B.2.37}$$

Finally, to ensure that all constant terms collectively equal zero, we need $\xi_0 = T_0/T_1$ where

$$\begin{aligned} T_0 &= [\xi_w \bar{w} + \xi_\pi \bar{\pi} + \xi_a \bar{a} + \xi_x \bar{x} + \xi_z \bar{z} + \xi_k \bar{k}] + \left(\frac{\phi \psi_K \bar{\Psi}}{\sigma \bar{K}} \right) \times \\ & [(\bar{w} + (\sigma - 1)(\bar{\pi} + \bar{a})) + (1 + u^{H,}) - u^{H,} \sigma \bar{x}] - \\ & u^{M,}(\theta - 1) \log \alpha + \upsilon \delta \left(\frac{1+\alpha}{\alpha} \right), \qquad \text{(B.2.38)} \\ T_1 &= u_2^{K,}(1 + \xi_k) + u_1^{K,}. \qquad \text{(B.2.39)} \end{aligned}$$

(B.2.31)-(B.2.39) then complete the specification of equilibrium k_{t+1} as a function of current state variables. It is also useful to re-express (B.2.15) in terms of $h_t = \log(H_t)$. Recalling that $\hat{H}_t = h_t - \bar{h}$, (B.2.15) gives

$$h_t - \bar{h} = (\nu_H)^{-1} \left[w_t + (\sigma - 1)(\pi_t + a_t + \psi_K k_t) - \sigma x_t - (\bar{w} + (\sigma - 1)(\bar{\pi} + \bar{a} + \psi_K \bar{k}) - \sigma \bar{x}) \right]. \qquad \text{(B.2.40)}$$

B.2.3 Equilibrium Industry Equity Risk Premium

Log-linearizing dividends in Equation (2.14) implies

$$\begin{aligned} \bar{D}(1 + \hat{D}_t) &= \bar{\Psi}(1 + \hat{\Psi}_t) - \bar{X}\bar{H}(1 + \hat{X}_t + \hat{H}_t) - [\bar{Z}\bar{I}(1 + \hat{Z}_t + \hat{I}_t) + \\ & 0.5\upsilon \left(\frac{(\bar{I})^2}{\bar{K}} \right) (1 + 2\hat{I}_t - \hat{K}_t)]. \qquad \text{(B.2.41)} \end{aligned}$$

Using the facts that $\bar{I} = \delta \bar{K}$, $\bar{D} = \bar{\Psi} - \bar{X}\bar{H} - \delta \bar{K}(\bar{Z} + 0.5\upsilon\delta)$, and substituting for $\hat{I}_t$ from (B.2.18) yields

$$\begin{aligned} \bar{D}\hat{D}_t &= \bar{\Psi}\hat{\Psi}_t - \bar{X}\bar{H}(\hat{X}_t + \hat{H}_t) - \bar{K}[\delta \bar{Z}\hat{Z}_t + \hat{K}_{t+1}(\bar{Z} + \upsilon\delta) + \\ & \hat{K}_t\{\bar{Z}(1 - \delta) + \left(\frac{\upsilon\delta}{2} \right)(2 - \delta)\}]. \qquad \text{(B.2.42)} \end{aligned}$$

Now, recall from (B.2.12), (B.2.15) (or (B.2.40)) that

$$\begin{aligned} \hat{\Psi}_t &= \left(\frac{\phi}{\sigma} \right) \left[\hat{W}_t + (\sigma - 1) \left(\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t + \psi_H \hat{H}_t \right) \right], \qquad \text{(B.2.43)} \\ \hat{H}_t &= (\nu_H)^{-1} \left[\hat{W}_t + (\sigma - 1) \left(\hat{P}_t + \hat{A}_t + \psi_K \hat{K}_t \right) - \sigma \hat{X}_t \right]. \qquad \text{(B.2.44)} \end{aligned}$$

But recognizing that $\hat{D}_t = d_t - \bar{d}$, substituting (B.2.43)-(B.2.44) and (B.2.29) (for $\hat{K}_{t+1} = k_{t+1} - k$) in (B.2.42), we can write the log of dividends as

$$d_t = N_{d,0} + N_{d,w} w_t + N_{d,\pi} \pi_t + N_{d,a} a_t + N_{d,x} x_t + N_{d,z} z_t + N_{d,k} k_t, \qquad \text{(B.2.45)}$$

where

$$N_{d,w} = (\bar{D})^{-1}\left[\frac{\bar{\Psi}\phi}{\sigma}\left(1+\frac{\psi_H(\sigma-1)}{\nu_H}\right)-\left(\frac{\bar{X}\bar{H}}{\nu_H}\right)-\bar{K}(\bar{Z}+\upsilon\delta)\xi_w\right] \tag{B.2.46}$$

$$N_{d,\pi} = (\bar{D})^{-1}\left[\frac{(\sigma-1)\bar{\Psi}\phi}{\sigma}\left(1+\frac{\psi_H(\sigma-1)}{\nu_H}\right)-\left(\frac{\bar{X}\bar{H}(\sigma-1)}{\nu_H}\right)-\right. \\ \left.\bar{K}(\bar{Z}+\upsilon\delta)\xi_\pi\right] \tag{B.2.47}$$

$$N_{d,a} = (\bar{D})^{-1}\left[\frac{(\sigma-1)\bar{\Psi}\phi}{\sigma}\left(1+\frac{\psi_H(\sigma-1)}{\nu_H}\right)-\left(\frac{\bar{X}\bar{H}(\sigma-1)}{\nu_H}\right)-\right. \\ \left.\bar{K}(\bar{Z}+\upsilon\delta)\xi_a\right] \tag{B.2.48}$$

$$N_{d,x} = -(\bar{D})^{-1}\left[\bar{\Psi}\phi\left(\frac{\psi_H}{\nu_H}\right)+\bar{X}\bar{H}+\bar{K}(\bar{Z}+\upsilon\delta)\xi_x\right] \tag{B.2.49}$$

$$N_{d,z} = -(\bar{D})^{-1}\bar{K}\left[\bar{Z}\delta+(\bar{Z}+\upsilon\delta)\xi_z\right] \tag{B.2.50}$$

$$N_{d,k} = (\bar{D})^{-1}\left[\frac{(\sigma-1)\psi_K\bar{\Psi}\phi}{\sigma}\left(1+\frac{\psi_H(\sigma-1)}{\nu_H}\right)-\left(\frac{\bar{X}\bar{H}(\sigma-1)\psi_K}{\nu_H}\right)-\right. \\ \left.\bar{K}(\bar{Z}+\upsilon\delta)\xi_k+\bar{K}\left\{\bar{Z}(1-\delta)+\left(\frac{\upsilon\delta}{2}\right)(2-\delta)\right\}\right], \tag{B.2.51}$$

and $N_{d,0}$ is a term of steady state constants that will not affect the covariance function of the ERP that we consider below.

With log equilibrium dividends in hand, we return to equilibrium equity return condition (2.7) to deduce the state representation of the log of the stock price $s_t = \log(S_t)$. We first rewrite the equilibrium condition as

$$\mathbb{E}_t\left[M_{t,t+1}\left(\frac{D_{t+1}}{S_t}+\frac{S_{t+1}}{S_t}\right)\right]=1. \tag{B.2.52}$$

Log-linearizing (B.2.52) we get (using $\bar{M}=\alpha$)

$$\alpha\mathbb{E}_t\left[\left(\frac{\bar{D}}{\bar{S}}\right)\left(1+\hat{M}_{t,t+1}+\hat{D}_{t+1}-\hat{S}_t\right)+\left(1+\hat{M}_{t,t+1}+\hat{S}_{t+1}-\hat{S}_t\right)\right]=1. \tag{B.2.53}$$

However, in the steady state $\bar{S}=\frac{\alpha\bar{D}}{1-\alpha}$, and hence

$$\alpha\left(\frac{\bar{D}}{\bar{S}}+1\right)=\alpha\left(\frac{1-\alpha}{\alpha}+1\right)=1. \tag{B.2.54}$$

Therefore, (B.2.53) becomes (recognizing $\left(\frac{\bar{D}}{\bar{S}}+1\right)=1/\alpha$)

$$\mathbb{E}_t\left[\hat{M}_{t,t+1}+(1-\alpha)\hat{D}_{t+1}+\alpha\hat{S}_{t+1}-\hat{S}_t\right]=0. \tag{B.2.55}$$

We conjecture that

$$s_t = N_{s,0} + N_{s,w} w_t + N_{s,\pi}\pi_t + N_{s,a} a_t + N_{s,x} x_t + N_{s,z} z_t + N_{s,k} k_t, \tag{B.2.56}$$

so that

$$\begin{aligned}\mathbb{E}_t\left[\alpha\hat{S}_{t+1} - \hat{S}_t\right] &= (\alpha-1)(N_{s,0} - \bar{s}) + N_{s,w}(\alpha\rho_w - 1)w_t + N_{s,\pi}(\alpha\rho_\pi - 1)\pi_t + \\ &\quad N_{s,a}(\alpha\rho_a - 1)a_t + N_{s,x}(\alpha\rho_x - 1)x_t + \\ &\quad N_{s,z}(\alpha\rho_z - 1)z_t + N_{s,k}(\alpha k_{t+1} - k_t). \end{aligned} \tag{B.2.57}$$

Meanwhile, from the foregoing

$$\begin{aligned}\mathbb{E}_t\left[(1-\alpha)\hat{D}_{t+1}\right] &= (1-\alpha)\left[N_{d,0} + N_{d,w}\rho_w w_t + N_{d,\pi}\rho_\pi\pi_t + N_{d,a}\rho_a a_t + \right. \\ &\quad \left. N_{d,x}\rho_x x_t + N_{d,z}\rho_z z_t + N_{d,k}k_{t+1} - \bar{d}\right], \end{aligned} \tag{B.2.58}$$

$$\mathbb{E}_t\left[\hat{M}_{t,t+1}\right] = (\theta-1)\log\alpha + B_w w_t + (B_\pi + (1-\rho_\pi))\pi_t. \tag{B.2.59}$$

But using (B.2.29), we have

$$\alpha k_{t+1} - k_t = \alpha\left[\xi_0 + \xi_w w_t + \xi_\pi\pi_t + \xi_a a_t + \xi_x x_t + \xi_z z_t\right] + (\alpha\xi_k - 1)k_t, \tag{B.2.60}$$

$$(1-\alpha)k_{t+1} = (1-\alpha)\left[\xi_0 + \xi_w w_t + \xi_\pi\pi_t + \xi_a a_t + \xi_x x_t + \xi_z z_t + \xi_k k_t\right]. \tag{B.2.61}$$

Thus substituting (B.2.60)-(B.2.61) in (B.2.57)-(B.2.58) we get (up to constants),

$$\mathbb{E}_t\left[\hat{M}_{t,t+1} + (1-\alpha)\hat{D}_{t+1} + \alpha\hat{S}_{t+1} - \hat{S}_t\right] =$$

$$\begin{aligned}0 &= w_t\left(B_w + N_{s,w}(\alpha\rho_w - 1) + (1-\alpha)N_{d,w}\rho_w + \xi_w(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + \pi_t\left(B_\pi + (1-\rho_\pi) + \right. \\ &\quad \left. N_{s,\pi}(\alpha\rho_\pi - 1) + (1-\alpha)N_{d,\pi}\rho_\pi + \xi_\pi(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + a_t\left(N_{s,a}(\alpha\rho_a - 1) + (1-\alpha)N_{d,a}\rho_a + \right. \\ &\quad \left. \xi_a(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + x_t\left(N_{s,x}(\alpha\rho_x - 1) + (1-\alpha)N_{d,x}\rho_x + \xi_x(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + \\ &\quad z_t\left(N_{s,z}(\alpha\rho_z - 1) + (1-\alpha)N_{d,z}\rho_z + \xi_z(\alpha N_{s,k} + (1-\alpha)N_{d,k})\right) + \\ &\quad k_t\left(N_{s,k}(\alpha\xi_k - 1) + (1-\alpha)N_{d,k}\xi_k\right). \end{aligned} \tag{B.2.62}$$

Hence, to ensure that items multiplying k_t in (B.2.63) are zero, we must have

$$N_{s,k} = \frac{(1-\alpha)N_{d,k}\xi_k}{1-\alpha\xi_k}, \tag{B.2.63}$$

where $N_{d,k}$ is computed in (B.2.50). With $N_{s,k}$ in hand, we can choose the remaining coefficients in (B.2.56) to ensure that (B.2.55) holds for all values of the state variables. That is,

$$
\begin{aligned}
N_{s,w} &= \frac{B_w + (1-\alpha)N_{d,w}\rho_w + \xi_w(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho_w}, \\
N_{s,\pi} &= \frac{B_\pi + (1-\rho_\pi) + (1-\alpha)N_{d,\pi}\rho_\pi + \xi_\pi(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho_\pi}, \\
N_{s,a} &= \frac{(1-\alpha)N_{d,a}\rho_a + \xi_a(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho_a}, \\
N_{s,x} &= \frac{(1-\alpha)N_{d,x}\rho_x + \xi_x(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho_x}, \\
N_{s,z} &= \frac{(1-\alpha)N_{d,z}\rho_z + \xi_z(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{1-\alpha\rho z}, \qquad \text{(B.2.64)}
\end{aligned}
$$

Finally, the constant term in (B.2.56) can be computed to be

$$
N_{s,0} = \frac{(\alpha-1)(N_{d,0} - \bar{d} + \bar{s} + N_{d,k}\xi_0) - \alpha N_{s,k}\xi_0 - (\theta-1)\log\alpha}{(\alpha-1)}. \qquad \text{(B.2.65)}
$$

The coefficients in (B.2.64)-(B.2.65) verify that s_t is indeed an affine function of the state variables.

Next, the Euler condition for equity returns imply (in the standard fashion)

$$
\mathbb{E}_t\left[R_{t+1} - R^f_{t+1}\right] = -Cov_t\left(M_{t,t+1}, \frac{D_{t+1} + S_{n,t+1}}{S_t}\right) R_{f,t+1}, \qquad \text{(B.2.66)}
$$

or dividing both sides of (B.2.66) by $R_{f,t+1}$

$$
\mathbb{E}_t\left[\frac{R_{t+1}}{R^f_{t+1}} - 1\right] = -Cov_t\left(M_{t,t+1}, \frac{D_{t+1} + S_{t+1}}{S_t}\right). \qquad \text{(B.2.67)}
$$

Log-linearizing the left hand side of (B.2.67) gives (for $r_{t+1} = \log(R_{t+1})$, $r^f_{t+1} = r^f_{t+1}$)

$$
\begin{aligned}
\mathbb{E}_t\left[\frac{R_{t+1}}{R^f_{t+1}} - 1\right] &= \mathbb{E}_t\left[\left(\frac{R}{R^f}\right)\left(1 + \hat{R}_{t+1} - \hat{R}^f_{t+1}\right) - 1\right] \\
&= \mathbb{E}_t\left[r_{t+1} - r^f_{t+1}\right],
\end{aligned}
$$

since in the deterministic steady state $\frac{R}{R^f} = 1$ and hence $\hat{R}_{t+1} - \hat{R}^f_{t+1} = r_{t+1} - r^f_{t+1}$. Then log-linearizing both sides of (B.2.67) yields

$$\begin{aligned}\mathbb{E}_t\left[r_{t+1}-r^f_{t+1}\right] &= -Cov_t\left(\alpha(1+\hat{M}_{t,t+1}),\frac{\bar{D}}{\bar{S}}(1+\hat{D}_{t+1})+(1+\hat{S}_{t+1}-\hat{S}_t\right)\\ &= -\alpha Cov_t\left(\hat{M}_{t,t+1},\frac{\bar{D}}{\bar{S}}\hat{D}_{t+1}+\hat{S}_{t+1}\right)\\ &= -\alpha Cov_t\left(\lambda_{t+1}-\pi_{t+1},\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}\right),\end{aligned} \tag{B.2.68}$$

where the final equality follows from recognizing that $\hat{M}_{t,t+1}=m_{t+1}-\log\alpha, m_{t+1}=\lambda_{t+1}+\pi_t-\pi_{t+1}, \hat{D}_{t+1}+\hat{S}_{t+1}=d_{t+1}+s_{t+1}-\log(\bar{D})-\log(\bar{S})$, and $\bar{S}=\frac{\alpha\bar{D}}{1-\alpha}$. But

$$-\alpha Cov_t\left(\lambda_{t+1}-\pi_{t+1},\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}\right)=\alpha Cov_t\left(\lambda_{t+1}-\pi_{t+1},-\left\{\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}\right\}\right)$$

$$=\alpha Cov_t\left(\lambda_{t+1}-\pi_{t+1}-\mathbb{E}_t[\lambda_{t+1}-\pi_{t+1}],-\left\{\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}-\mathbb{E}_t[\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}]\right\}\right). \tag{B.2.69}$$

From (B.2.4) and (B.2.46)-(B.2.65) we get (recognizing that k_{t+1} is deterministic conditional on Γ_t)

$$\lambda_{t+1}-\pi_{t+1}-\mathbb{E}_t[\lambda_{t+1}-\pi_{t+1}]=b_w\varepsilon^w_{t+1}+(b_\pi-1)\varepsilon^\pi_{t+1}, \tag{B.2.70}$$

$$-\left\{\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}-\mathbb{E}_t\left[\left(\frac{1-\alpha}{\alpha}\right)d_{t+1}+s_{t+1}\right]\right\}=-\left(\left(\frac{1-\alpha}{\alpha}\right)\mathbf{N}_d+\mathbf{N}_s\right)'\boldsymbol{\varepsilon}_{t+1}, \tag{B.2.71}$$

where b_w and b_π are defined in (B.2.5). Meanwhile, $\mathbf{N}_d=(N_{d,w},N_{d,\pi},N_{d,a},N_{d,x},N_{d,z})$ are defined in (B.2.46)-(B.2.51) and $\mathbf{N}_s=(N_{s,w},N_{s,\pi},N_{s,a},N_{s,x},N_{s,z})$ are defined in (B.2.64). An additional piece of notation allows one to more concisely express (B.2.69). Put

$$\tilde{N}_w=-\left[\left(\frac{1-\alpha}{\alpha}\right)N_{d,w}+N_{s,w}\right],$$

and similarly for $\tilde{N}_\pi,\tilde{N}_a,\tilde{N}_x$ to get from (B.2.46)-(B.2.51) and (B.2.64) (upon switching the sign of the denominator)

$$\tilde{N}_w = \frac{(1-\alpha)N_{d,w} + \alpha(B_w + \xi_w(\alpha N_{s,k} + (1-\alpha)N_{d,k}))}{\alpha(\alpha\rho_w - 1)}, \quad \text{(B.2.72)}$$

$$\tilde{N}_\pi = \frac{(1-\alpha)N_{d,\pi} + \alpha(B_\pi + (1-\rho_\pi) + \xi_\pi(\alpha N_{s,k} + (1-\alpha)N_{d,k}))}{\alpha(\alpha\rho_\pi - 1)}, \quad \text{(B.2.73)}$$

$$\tilde{N}_a = \frac{(1-\alpha)N_{d,a} + \xi_a(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{\alpha(\alpha\rho_a - 1)}, \quad \text{(B.2.74)}$$

$$\tilde{N}_x = \frac{(1-\alpha)N_{d,x} + \xi_x(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{\alpha(\alpha\rho_x - 1)}, \quad \text{(B.2.75)}$$

$$\tilde{N}_z = \frac{(1-\alpha)N_{d,z} + \xi_x(\alpha N_{s,k} + (1-\alpha)N_{d,k})}{\alpha(\alpha\rho_z - 1)}. \quad \text{(B.2.76)}$$

Then,

$$\alpha Cov_t\left(\lambda_{t+1} - \pi_{t+1}, -\left\{\left(\frac{1-\alpha}{\alpha}\right) d_{t+1} + s_{t+1})\right\}\right) =$$

$$b_w\tilde{N}Var_t(\varepsilon^w_{t+1}) + (b_\pi - 1)\tilde{N}_\pi Var_t(\varepsilon^\pi_{t+1}) + ((b_\pi - 1)\tilde{N}_w + b_w\tilde{N}_\pi)Cov_t(\varepsilon^w_{t+1}, \varepsilon^\pi_{t+1}) +$$
$$b_w\tilde{N}_a Cov_t(\varepsilon^w_{t+1}, \varepsilon^a_{t+1}) + (b_\pi - 1)\tilde{N}_a Cov_t(\varepsilon^\pi_{t+1}, \varepsilon^a_{t+1}) + b_w\tilde{N}_x Cov_t(\varepsilon^w_{t+1}, \varepsilon^x_{t+1}) + (b_\pi - 1) \times$$
$$\tilde{N}_x Cov_t(\varepsilon^\pi_{t+1}, \varepsilon^x_{t+1}) + b_w\tilde{N}_z Cov_t(\varepsilon^w_{t+1}, \varepsilon^z_{t+1}) + (b_\pi - 1)\tilde{N}_z Cov_t(\varepsilon^\pi_{t+1}, \varepsilon^z_{t+1}) \quad \text{(B.2.77)}$$

Hence, we can write

$$\mathbb{E}_t[r_{t+1} - r^f_{t+1}] = \beta_w\Phi^2_w + \beta_\pi\Phi^2_\pi + \beta_{w\pi}\Phi_{w\pi} + \beta_{aw}\Phi_{aw} + \beta_{a\pi}\Phi_{a\pi} + \beta_{xw}\Phi_{xw} + \beta_{x\pi}\Phi_{x\pi} + \beta_{zw}\Phi_{zw} + \beta_{z\pi}\Phi_{z\pi},$$

where

$$\beta_w = \alpha b_w\tilde{N}_w, \beta_\pi = \alpha(b_\pi - 1)\tilde{N}_\pi, \beta_{w\pi} = \alpha((b_\pi - 1)\tilde{N}_w + b_w\tilde{N}_\pi),$$
$$\beta_{aw} = \alpha b_w\tilde{N}_a, \beta_{a\pi} = \alpha(b_\pi - 1)\tilde{N}_a, \beta_{xw} = \alpha b_w\tilde{N}_x,$$
$$\beta_{x\pi} = \alpha(b_\pi - 1)\tilde{N}_x, \beta_{zw} = \alpha b_w\tilde{N}_z, \beta_{z\pi} = \alpha(b_\pi - 1)\tilde{N}_z \quad \text{(B.2.78)}$$

C.1 Steady State Parameterization

To match the endogenous $\bar{H}, \bar{K}$ and $\bar{Y}$ to the sample means of the industry data (in per capita terms), we proceed as follows. Let MC, Y_{data} denote the sample mean of the materials cost. Then, by definition, $MC = \bar{X}\bar{H}$. Hence, the materials input optimality condition (B.1.1) can be written (recognizing that $\psi_H\bar{Y}/\bar{H}$)

$$\frac{\bar{p}\bar{Y}}{\bar{H}} = \frac{MC}{\bar{H}}, \quad \text{(C.1.1)}$$

which implies that

$$\psi_H(\bar{p}\bar{Y}) = MC. \tag{C.1.2}$$

Meanwhile, the steady state Euler condition (B.1.2) implies

$$\psi_K\left(\frac{\bar{p}\bar{Y}}{\bar{K}}\right) = \left(\frac{\bar{Z}+\upsilon\delta}{\alpha}\right)(1-\alpha(1-\delta)) - 0.5\upsilon(\delta)^2. \tag{C.1.3}$$

Put $e_K \equiv \left(\frac{\bar{Z}+\upsilon\delta}{\alpha}\right)(1-\alpha(1-\delta)) - 0.5\upsilon(\delta)^2$. Now dividing (C.1.2) by (C.1.3) gives

$$\bar{K} = \left(\frac{\psi_K}{\psi_H}\right)\left[\frac{MC}{e_K}\right]. \tag{C.1.4}$$

Then we set

$$\bar{H} = \left(\frac{Y_{data}}{(\bar{K})^{\psi_K}}\right)^{1/\psi_H}, \tag{C.1.5}$$

where $\bar{K}$ is given in (C.1.4). $\bar{X}$ is then computed as the ratio $MC/\bar{H}$. And using $\bar{H}$ and $\bar{K}$ above, $\bar{A}$ is set such that

$$\bar{A} = \frac{Y_{data}}{(\bar{K})^{\psi_K}(H)^{\psi_H}}. \tag{C.1.6}$$

Finally, $\bar{Z}$, ψ_K, ψ_H are calibrated to match $\bar{H}, \bar{K}$ and $\bar{Y}$ with the sample mean values of industry data (in per capita terms).

Table A.1. Steady State Values for Numerical Calculations

Steady State Values			
Aggregate	$\bar{W}$ (dollars)	$\bar{P}$	$\frac{\bar{C}}{\bar{W}}$
	19905	1.286	0.78 (0.78)
Sector	$\bar{Y}$ (million dollars)	$\bar{X}\bar{H}$(million dollars)	$\bar{Z}\bar{K}$ (million dollars)
Consumer Durables	18.23 (18.23)	9.56 (9.56)	10.38 (10.34)
Consumer Non-Durables	23.69 (23.69)	13.96 (13.96)	12.48 (12.71)

Notes to Table: This table displays the steady state aggregate and sectoral values used in the numerical calculations of sectoral equity risk premium (ERP) in Table 1. The steady state value for per capita aggregate personal income ($\bar{W}$) is the sample mean, and $\bar{P}$ is the sample mean of the per capita consumption-to-income ratio C/W. The sectoral production variables are the steady state values computed from the model, while the entries in parentheses are the corresponding sample means.